Pragmatics inferences of modified numerals and modals

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1 The data

Class B numeral modifiers: numeral modifiers that give rise to ignorance effects in the absence of an obviating operator (Nouwen, 2010)

(1) a. I know exactly how many books there are in this bookshop, #and it's { at least / at most } 10,000. *class B*
b. I know exactly how many books there are in this bookshop, and it's { more than / fewer than } 10,000. *class A*

This study is about sentences where class B numeral modifiers occur with modals. The two most natural combinations are *at least* + $\Box$ and *at most* + $\Diamond$:

(2) a. You're required to write at least three pages.
b. You're allowed to write at most three pages.

Two readings of (2-a):

1. *Authoritative reading*: is required that you write no fewer than three pages — you write three or more pages in all possible worlds.

2. *Ignorance reading*: I don't know what the minimum number of pages you are required to write is, but I know the minimum is three or higher:

Two readings of (2-b):

1. *Authoritative reading*: you're not allowed to write more than three pages — there is no possible world in which you write more than three pages

2. *Ignorance reading*: I don't know what the maximum number of pages you are required to write is, but I know the maximum is three or lower

The other two possible combinations are given in (3).

(3) a. You're required to write at most three pages.
b. You're allowed to write at least three pages.

Here the ignorance readings are more prevalent.

- For (3-a): I don't know what the minimum number of pages you are required to write is, but I know the minimum is three or lower
- For (3-b): I don't know what the maximum number of pages you are allowed to write is, but I know the maximum is three or lower

(3-a) also has an authoritative reading similar to (2-b), though (2-b) is preferred to express this meaning

Summary of the data

- Without a modal, class B numeral modifiers give rise to ignorance effects
- When a class B modifier is combined with its 'natural partner' — *at least* + $\Box$ and *at most* + $\Diamond$ — the authoritative reading is the most prevalent one
- When a class B modifier is combined with the other partner, the ignorance reading is the most prevalent one. For *at least* + $\Diamond$ this appears to be the only reading, while for *at most* + $\Box$ there is also an authoritative reading

Roadmap

Section 2 Previous accounts
Section 3 Inquisitive semantics and epistemic inferences
Section 4 Analysis
Section 5 Authoritative readings with *at most* and universal modals
Section 6 A note on the nature of epistemic inferences
Section 7 Conclusion

2 Previous accounts: Schwarz (2013)

Goal Account for ignorance effects of class B modifiers in unembedded contexts and for the two readings that arise when they interact with modals

Method Quantity implicatures

Denotations:

(4) a. $[\![\text{at least}]\!] = \lambda d_d \lambda P_{\langle d,t \rangle}.\text{MAX}\{n \mid P(n)\} \geq d$
b. $[\![\text{at most}]\!] = \lambda d_d \lambda P_{\langle d,t \rangle}.\text{MAX}\{n \mid P(n)\} \leq d$

where $\text{MAX}(P) = \iota n \,.\, P(n) \wedge \forall n' \, [P(n') \rightarrow n' \leq n]$ (Heim, 2000)

Horn sets:

(5) a. { 1, 2, 3, 4, 5, ... }
b. { at least, exactly, at most }

(6) $[\![\text{exactly}]\!] = \lambda d_d \lambda P_{\langle d,t\rangle}.\text{MAX}\{n \mid P(n)\} = d$

Interactions with modals: the two readings (authoritative vs. ignorance) are derived using scope:

- Narrow scope for the modified numeral is supposed to generate an authoritative reading
- Wide scope for the modified numeral is supposed to generate an ignorance reading

2.1 Universal modals

(7) Mary is required to write at least five pages.

(8) a. $\Box$ [MAX { n | Mary writes n pages } $\geq$ 5]
b. MAX { n | $\Box$ [Mary writes n pages] } $\geq$ 5

Stronger alternatives to (8-a):

(9) a. $\Box$ [MAX { n | Mary writes n pages } = 5]
b. $\Box$ [MAX { n | Mary writes n pages } $\geq$ 6]

Primary implicatures:

(10) a. $\neg$B $\Box$ [MAX { n | Mary writes n pages } = 5]
b. $\neg$B $\Box$ [MAX { n | Mary writes n pages } $\geq$ 6]
where 'B' stands for 'the speaker believes that'

Secondary implicatures:

(11) a. B $\neg\Box$ [MAX { n | Mary writes n pages } = 5]
b. B $\neg\Box$ [MAX { n | Mary writes n pages } $\geq$ 6]

The assertion in (8-a) combination with these secondary implicatures yields the authoritative reading: the speaker believes Mary is required to write five or more pages but she is not required to write exactly five and she is not required to write six or more.

The wide scope reading in (8-b) has the stronger alternatives in (12):

(12) a. MAX { n | $\Box$ [Mary writes n pages] } = 5
b. MAX { n | $\Box$ [Mary writes n pages] } $\geq$ 6

These alternatives are symmetric: negating both alternatives yields a contradiction to the assertion (Sauerland, 2004). As a result, the primary implicatures cannot be strengthened to secondary implicatures.

Primary implicatures:

(13) a. $\neg$ B $\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} = 5$
b. $\neg$ B $\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} \geq 6$

Impossible secondary implicatures:

(14) a. B $\neg$ $\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} = 5$
b. B $\neg$ $\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} \geq 6$

From the combination of the assertion and the primary implicatures, we can conclude (15).

(15) a. P $\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} = 5$
(follows from (8-b) and (13-b))
b. P $\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} \geq 6$
(follows from (8-b) and (13-a))
where 'P' stands for 'the speaker considers it possible that'

The speaker considers both (15-a) and (15-b) possible, so we can conclude that she does not know which of these options is true and derive ignorance implicatures:

(16) a. ?$\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} = 5$
b. ?$\textsc{max}$ $\{ n \mid \Box$ [Mary writes n pages] $\} \geq 6$
where '?' stands for 'the speaker does not know whetther'

Ignorance reading: The speaker believes that Mary is required to write at least five pages, but she is not sure whether Mary is required to write exactly five pages and she is not sure whether Mary is required to write at least six pages.

The combination of *at most* and a universal modal works the same way.

How the account is supposed to work:

- Narrow scope for the modified numeral → no symmetry → secondary/scalar implicatures → authoritative reading
- Wide scope for the modified numeral → symmetry → no secondary implicatures → ignorance implicatures → ignorance reading

2.2 Existential modals — *at least*

(17) Mary is allowed to write at least five pages.

(18) a. $\Diamond$ [$\textsc{max}$ $\{ n \mid$ Mary writes n pages $\} \geq 5$]
b. $\textsc{max}$ $\{ n \mid \Diamond$ [Mary writes n pages] $\} \geq 5$

Stronger alternatives to (18-a):

(19) a. $\Diamond$ [$\textsc{max}$ { n | Mary writes n pages } $= 5$]
b. $\Diamond$ [$\textsc{max}$ { n | Mary writes n pages } ≥ 6]

Primary implicatures:

(20) a. $\neg$B $\Diamond$ [$\textsc{max}$ { n | Mary writes n pages } $= 5$]
b. $\neg$B $\Diamond$ [$\textsc{max}$ { n | Mary writes n pages } ≥ 6]

The stronger alternatives are symmetric, so ignorance implicatures are derived:

(21) a. ?$\Diamond$ [$\textsc{max}$ { n | Mary writes n pages } $= 5$]
b. ?$\Diamond$ [$\textsc{max}$ { n | Mary writes n pages } ≥ 6]

Narrow scope reading: the speaker believes that Mary is allowed to write five or more pages, but she is not sure whether Mary is allowed to write exactly five pages and she is not sure whether Mary is allowed to write six or more pages → **not attested**[1]
Stronger alternatives to (18-b):

(22) a. $\textsc{max}$ { n | $\Diamond$ [Mary writes n pages] } $= 5$
b. $\textsc{max}$ { n | $\Diamond$ [Mary writes n pages] } ≥ 6

Symmetry → ignorance implicatures:

[1]The reading where *at least* takes scope under an existential modal is not attested

(i) has the two theoretically possible readings in (ii).

(i) Marin is allowed to read at least five books.

(ii) a. $\Diamond$ [*max* { n | Marin reads n books } ≥ 5]
b. *max* { n | $\Diamond$ [Marin reads n books] } ≥ 5

Difference: only the inverse scope reading carries the presupposition that there is an upper bound to what is allowed

- (ii-a) merely conveys that there is a permissible world where Marin reads five or more books
- (ii-b) expresses that there is a maximum number of books Marin is allowed to read, and that maximum is five or higher

Scenario: Marin is a school child in a school with the following rules. The children in Year 4 are allowed to read as many books as they like during the school year. The children in Year 5, on the other hand, are expected to focus more on subjects such as maths and geography, and they have an upper limit to the number of books they can read at school. The exact upper limit varies from child to child and depends on the child's reading level and the child's grades for other subjects. In addition, new research has just been published that indicates that children who read 20 books a year or more have better vocabulary than those who read fewer than twenty books a year. One day, Marin's dad and another parent, David are talking about this new research. David wonders about Marin's vocabulary and asks the question in (iii).

(iii) David: Is Marin in Year 4 or in Year 5?

If she is in year five, there is an upper bound to the number of books she can read, and both answers below would be felicitous:

(iv) a. Marin is in Year 5. She is allowed to read at least twenty books.
b. Marin is in Year 5. She is allowed to read more than twenty books.

If she is in year four, only (v-b) is felicitous

(v) a. Marin is in Year 4. #She is allowed to read at least twenty books.
b. Marin is in Year 4. She is allowed to read more than twenty books.

(23) a. ?$\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} = 5$
b. ?$\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} \geq 6$

Wide scope reading: the speaker believes that the maximum number of pages Mary is allowed to write is five or more, but she is not sure if the maximum is exactly five or higher than five.

Problem As Schwarz admits, the narrow scope reading appears not to be there: (17) seems to convey that there is an upper bound. Schwarz claims that this weaker narrow scope reading is not visible because it is blocked by the stronger wide scope reading.

2.3 Existential modals — *at most*

(24) Mary is allowed to write at most five pages.

(25) a. $\Diamond\ [\ \text{MAX}\ \{\ n \mid \text{Mary writes } n \text{ pages}\ \} \leq 5\ ]$
b. $\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} \leq 5$

Stronger alternatives to (25-a):

(26) a. $\Diamond\ [\ \text{MAX}\ \{\ n \mid \text{Mary writes } n \text{ pages}\ \} = 5\ ]$
b. $\Diamond\ [\ \text{MAX}\ \{\ n \mid \text{Mary writes } n \text{ pages}\ \} \leq 4\ ]$

Symmetry → ignorance implicatures:

(27) a. ?$\Diamond\ [\ \text{MAX}\ \{\ n \mid \text{Mary writes } n \text{ pages}\ \} = 5\ ]$
b. ?$\Diamond\ [\ \text{MAX}\ \{\ n \mid \text{Mary writes } n \text{ pages}\ \} \leq 4\ ]$

Narrow scope reading: the speaker believes that Mary is allowed to write five or fewer pages but she is not sure whether Mary is allowed to write exactly five pages and she is not sure whether Mary is allowed to write four or fewer pages → **not attested**:

(28) a. You're allowed to take at most two biscuits, #and/but you can also take more.
b. You're allowed to take fewer than three biscuits, and/but you can also take more.

Stronger alternatives to (25-b):

(29) a. $\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} = 5$
b. $\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} \leq 4$

Symmetry → ignorance implicatures:

(30) a. ?$\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} = 5$
b. ?$\text{MAX}\ \{\ n \mid \Diamond\ [\ \text{Mary writes } n \text{ pages}\ ]\ \} \leq 4$

Wide scope reading: the speaker believes that the maximum number of pages Mary is allowed to write is five or fewer, but she is not sure if the maximum is exactly five or lower than five.

Overview of readings this account generates with existential modals:

At least + $\Diamond$	*At most* + $\Diamond$
• 2 ignorance readings • Wide scope ignorance reading: attested • Narrow scope ignorance reading: **not attested**	• 2 ignorance readings • Wide scope ignorance reading: attested • Narrow scope ignorance reading: **not attested** • Reading this account does not generate: authoritative reading with a strong upper bound

Table 1: Readings & problems for Schwarz (2011)

Three problems

1. The readings where *at least* and *at most* take scope under an existential modal are not attested
2. Existential modals always lead to symmetric alternatives, so even if these readings were attested, it is not possible to derive an authoritative reading for *at most* + an existential modal
3. The account disregards the fact that certain combinations give rise to authoritative readings while others do not, and attempts to derive both authoritative and epistemic readings for all possible combinations of modals and modified numerals

Schwarz links the pragmatic ambiguity to scope. Other accounts do this too (e.g. Kennedy, 2015a; Coppock & Brochhagen, 2013) and therefore run into similar problems with existential modals

Assumption (Blok, 2015a, 2019): class B numeral modifiers must take scope over existential modals

Where we are

- Class B numeral modifiers must take scope over existential modals
- This is why using scope to derive the ambiguity of sentences with modified numerals and modals does not work
- I will use the wide scope configuration as a basis for my account, using a mechanism of optional flattening rather than scope to get the two observed readings
- The framework I will use for my analysis is the framework of inquisitive semantics

3 Inquisitive semantics and epistemic inferences

Inquisitive semantics has been used in the literature on modified numerals to calculate epistemic inferences (Coppock & Brochhagen, 2013; Blok, 2015b, 2016; Ciardelli, Coppock, & Roelofsen, 2016; Blok, 2017; Cremers, Coppock, Dotlacil, & Roelofsen, 2017).

The epistemic inferences of class B numeral modifiers are said to be quality implicatures:

(31) The Maxim of Quality in Inquisitive Semantics (Ciardelli et al., 2016)
  a. $s \subseteq \text{info}(\phi)$
  b. if ϕ is inquisitive, then $s \notin [\![\phi]\!]$

The Maxim of Quality is only satisfied if both conditions are met. info(ϕ) is the information contained in a proposition ϕ: the union of all its possibilities. s is the speaker's information state: a set of worlds. (42-a) says that the speaker's information state must be a subset of the informative content of the proposition she utters.
The second part only comes into play when an inquisitive proposition is used. In this case, the speaker's information state cannot be an element of the proposition she utters. That is, none of the possibilities in the proposition can be the speaker's information state.

Ciardelli et al. (2016): class B numeral modifiers such at *at least* and *at most* give rise to inquisitive propositions containing two possibilities. For instance, (32) contains the two possibilities illustrated in (33): the possibility that Anne speaks exactly two languages, represented by p_2, and the possibility that she speaks three or more languages, represented by $p_{[3-\infty)}$

(32) Anne speaks at least two languages.

(33) $\{p_2, p_{[3-\infty)}\}$

Assuming that the speaker is being cooperative and following the Quality Maxim, we can conclude from the fact that she used an inquisitive proposition that she does not know which of the possibilities in the proposition are true. This is how the epistemic implicature comes about

4 Analysis

4.1 The basics

Lexical entries:[2]

[2] I assume here that *at least* and *at most* take possibilities as arguments. In reality, I believe that their type is flexible. One way to implement this is to follow Coppock and Brochhagen (2013) and assume flexible lexical entries, as in (i)-(ii), where α stands for any type ending in p and β is whatever type α takes as an argument.

(i) $[\![\text{at least}]\!]^{S,AL} = \{\lambda\alpha\lambda\beta.\text{MAX}_{AL}\ (\alpha(\beta))\ ,\ \lambda\alpha\lambda\beta.\cup\{\text{MAX}_{AL}\ p' \mid p' \ >_S \alpha(\beta)\}\}$

(34) $[\![\text{at least}]\!]^{S,AL} = \{\lambda p.\text{MAX}_{AL}\ p\ ,\ \lambda p. \cup \{\text{MAX}_{AL}\ p' \mid p'\ >_S p\}\}$

(35) $[\![\text{at most}]\!]^{S,AL} = \{\lambda p.\text{MAX}_{AL}\ p\ ,\ \lambda p. \cup \{\text{MAX}_{AL}\ p' \mid p'\ <_S p\}\}$

(36) $\text{MAX}_{AL} = \lambda p.\{w|w \in p \wedge \neg\exists p'[p' >_{AL} p \wedge w \in p']\}$

- Disjunctive: union of two possibilities
- First possibility: $\text{MAX}_{AL}\ p$, where p is the prejacent.
- MAX_{AL} takes a possibility and returns a subset of this possibility. None of the worlds in this subset are in a possibility p' that is ranked higher than p on a scale of alternatives AL
- Second possibility: union of the set of possibilities $\text{MAX}_{AL}\ p'$ for all possibilities p' that are ranked higher on S than p (*at least*) or all possibilities p' that are ranked lower on S than p (*at most*)
- S is an ordered version of CQ, a set of alternatives derived from the Rooth-Hamblin alternatives of a sentence using the Focus Principle (Beaver & Clark, 2008). For the purposes of this talk, it suffices to assume that $CQ = [\![\alpha]\!]^A$ (and therefore S is an ordered version of $[\![\alpha]\!]^A$)
- AL is a pragmatic scale of alternatives that is derived independently of the focus alternatives. In principle, $AL = S$, but when operations are applied to $[\![\alpha]\!]^A$, this affects S but not AL

Example with *at least*:

(37) Abdullah ate at least two sandwiches.

Prejacent:

(38) $[\![\text{Abdullah ate [two]}_F \text{ sandwiches}]\!]^O = p_2 = \{\exists x[\#x = 2 \wedge \text{sandwiches}(x) \wedge \text{ate}(\text{Abdullah}, x)]\}$

Scale of alternatives:

(ii) $[\![\text{at most}]\!]^{S,AL} = \{\lambda\alpha\lambda\beta.\text{MAX}_{AL}\ (\alpha(\beta))\ ,\ \lambda\alpha\lambda\beta. \cup \{\text{MAX}_{AL}\ p' \mid p'\ <_S \alpha(\beta)\}\}$

This way, *at least* and *at most* can be interpreted *in situ*. For instance, in (iii) *at least* takes *five beers* as an argument, which is turned into a regular quantifier over individuals through Hackl's (2000) *many* quantifier. The definition of *five* many *beers* is given in (iv).

(iii) Indira drank at least five beers.

(iv) $[\![\text{five } many \text{ beers}]\!] = \{\ \lambda P_{\langle e,p\rangle}.\exists x_e[\#x = 5 \wedge \text{beers}(x) \wedge P(x)]\ \}$

The relevant denotation of *at least* is the one in (v). Applying *at least* to *five beers* yields (vi), which can then be combined with *drank* using QR or type shifting.

(v) $[\![\text{at least}]\!]^{S,AL} = \{\lambda \mathcal{P}_{\langle\langle e,p\rangle,p\rangle}\lambda Q_{\langle e,p\rangle}.\text{MAX}_{AL}\ (\mathcal{P}(Q))\ ,\ \lambda \mathcal{P}_{\langle\langle e,p\rangle,p\rangle}\lambda Q_{\langle e,p\rangle}. \cup \{\text{MAX}_{AL}\ p' \mid p'\ >_S \mathcal{P}(Q)\}\}$

(vi) $[\![\text{at least five } many \text{ beers}]\!] = \{\lambda Q_{\langle e,p\rangle}.\text{MAX}_{AL}\ (\exists x_e[\#x = 5 \wedge \text{beers}(x) \wedge Q(x)]), \lambda Q_{\langle e,p\rangle}. \cup \{\text{MAX}_{AL}\ p' \mid p'\ >_S \exists x_e[\#x = 5 \wedge \text{beers}(x) \wedge Q(x)]\}\}$

(39) $S = AL = p_0 < p_1 < p_2 < p_3 < p_4$ ...

Semantics of (37)

(40) $\{p_2 \wedge \neg p_3, p_3\}$

- First possibility: $\textsc{max}_{AL} = p_2 \wedge \neg p_3$
- Second possibility: result of applying $\textsc{max}_{AL}$ to all possibilities p' in S that are ordered higher than p:

  (41) $\textsc{max}_{AL}\ p_3 = p_3 \wedge \neg p_4$,
  $\textsc{max}_{AL}\ p_4 = p_4 \wedge \neg p_5$,
  $\textsc{max}_{AL}\ p_5 = p_5 \wedge \neg p_6$,
  etc.
  $= p_3$

Ignorance effects are derived using the Quantity Maxim enriched with an inquisitive part:

(42) The Maxim of Quality in Inquisitive Semantics (Ciardelli et al., 2016):
  a. $s \subseteq \text{info}(\phi)$
  b. if ϕ is inquisitive, then $s \notin [\![\phi]\!]$

  where s is the speaker's information state (a set of worlds), ϕ is a proposition, and $\text{info}(\phi)$ is the information contained in ϕ (the union of its possibilities)

- Assuming that the speaker is being cooperative and following the Quality Maxim, we can conclude from the fact that she used an inquisitive proposition that she does not know which of the possibilities in the proposition are true
- Given that *at least* and *at most* give rise to inquisitive propositions, they generate ignorance effects through the Maxim of Quality

Example with *at most*:

(43) Abdullah ate at most two sandwiches.

Applying *at most* to the prejacent in (38) derives:

(44) $\{p_2 \wedge \neg p_3, \{w_{[0]}, w_{[1]}\}\}$

where $w_{[n]}$ stands for the world in which Abdullah ate exactly n sandwiches

- We get $p_2 \wedge \neg p_3$ from $\textsc{max}_{AL}\ p_2$, as above
- We apply $\textsc{max}_{AL}$ to all propositions that are ordered lower than p_2 on S:(45):

(45) $\text{MAX}_{AL}\ p_0 = p_0 \wedge \neg p_1$,
$\text{MAX}_{AL}\ p_1 = p_1 \wedge \neg p_2$

The union of these two possibilities is $\{w_{[0]}, w_{[1]}\}$

4.2 Deriving epistemic and variation readings

Two ingredients:

1. Syntactic structure
2. Optional flattening

Syntactic structure

(46) Malika is { allowed / required } to adopt { at least / at most } two cats.

(47) [{ at least / at most } [{□/◊} [Malika adopts [[~~{ at least / at most }~~ [[[two]$_F$ *many*] cats]]]]]]

where:

(48) $[\![many]\!] = \{\lambda d_d \lambda P_{\langle e,t\rangle} \lambda Q_{\langle e,t\rangle}.\exists x[\#x = d \wedge P(x) \wedge Q(x)]\}$

The numeral modifier takes wide scope. Recall from the previous section that this must be the case when these modifiers occur with existential modals

Optional flattening

Coppock and Brochhagen (2013), inspired by Kratzer and Shimoyama (2002), assume that a modal flattens the set of possibilities in its scope. When the prejacent of a modal contains multiple possibilities, the modal returns the union of these possibilities:[3]

(49) $\Box >$ *at least 2* $\rightarrow \{\Box \cup \{p_2, p_3, p_4, \ldots\}\} = \{\Box p_2\}$

My proposal is to make this mechanism optional:

(50) **Optionality**
Modals optionally flatten both $[\![\alpha]\!]^O$ and $[\![\alpha]\!]^A$

a. When a modal takes scope under the modified numeral, flattening $[\![\alpha]\!]^O$ is vacuous (because its prejacent will already be flat) but flattening $[\![\alpha]\!]^A$ has an effect

b. When a universal modal takes scope over the modified numeral, flattening $[\![\alpha]\!]^A$ is vacuous (because the numeral modifier has already used the alternatives at this point) but flattening $[\![\alpha]\!]^O$ has an effect

[3]For them, this mechanism is linked to scope. They assume that all possible scope configurations between modified numerals and modals are possible, and the modal can flatten only when it takes wide scope.

Natural combinations

(51) Malika is required to adopt at least two cats.

Option 1: no flattening

(52) $[\![\text{Malika is required to adopt two cats}]\!]^O = \{\Box p_2\}$

Alternatives:

(53) $[\![\text{Malika is required to adopt two cats}]\!]^A = \{\Box p_0, \Box p_1, \Box p_2, \Box p_3, ...\}$

As mentioned above, in this case and all other cases in this handout, the CQ is equivalent to the set of Rooth-Hamblin alternatives $[\![\alpha]\!]^A$:

(54) $CQ = [\![\text{Malika is required to adopt two cats}]\!]^A = \{\Box p_0, \Box p_1, \Box p_2, \Box p_3, ...\}$

Ordering of the alternatives:

(55) $S = AL = \Box p_0 < \Box p_1 < \Box p_2 < \Box p_3 < \Box p_4$ ...

Applying *at least* to (52):

(56) $\{\Box p_2 \wedge \neg\Box p_3,$
$\cup \{\Box p_3 \wedge \neg\Box p_4, \Box p_4 \wedge \neg\Box p_5, ...\}\}$

Two possibilities:

- $\Box p_2 \wedge \neg\Box p_3$, is obtained simply by applying MAX_{AL} to $\Box p_2$. It says that Malika adopts two or more cats in every world, but she does not adopt three or more cats in every world. In other words: two cats is sufficient; three cats is not required
- $\{\Box p_3 \wedge \neg\Box p_4, \Box p_4 \wedge \neg\Box p_5, ...\}$ is obtained by applying MAX_{AL} to all higher alternatives, and this set of possibilities is turned into a set of worlds by applying the union operation

Inquisitive proposition → epistemic inferences: the speaker is not sure if Malika has to adopt two cats and for all numbers above two, the speaker is also not sure is Malika has to adopt that many cats

Option 2: flattening

Prejacent of the modal:

(57) $[\![\text{Malika adopts two cats}]\!]^O = \{p_2\}$

This is not an inquisitive proposition, so $[\![\alpha]\!]^O$ is already a singleton set: there is nothing there for the modal to flatten. But assuming that the numeral is the focused element in the sentence, $[\![\alpha]\!]^A$ contains multiple possibilities:

(58) $[\![\text{Malika adopts two cats}]\!]^A = \{p_0, p_1, p_2, ...\}$

The modal flattens this set:

(59) $[\![\text{Malika adopts two cats}]\!]^A = \{\{w_{[0]}, w_{[1]}, w_{[2]}, ...\}\} = \{p_0\}$

Adding the lexical meaning of the modal yields the ordinary meaning in (60) and the alternatives in (61).

(60) $[\![\text{Malika is required to adopt two cats}]\!]^O = \{\Box p_2\}$

(61) $[\![\text{Malika is required to adopt two cats}]\!]^A = \{\Box p_0\}$

The CQ is equivalent to $[\![\alpha]\!]^A$:

(62) $\text{CQ} = [\![\alpha]\!]^A = \{\Box p_0\}$

The set AL, on the other hand, is independent from $[\![\alpha]\!]^A$ and therefore stays as it is. Thus, we have:

(63) $S = \Box p_0$

(64) $AL = \Box p_0 < \Box p_1 < \Box p_2 < \Box p_3 < \Box p_4$

Adding *at least* yields:

(65) $\{\Box p_2 \wedge \neg \Box p_3\}$

This meaning comes about as follows:

- We apply MAX_{AL} to the prejacent $\Box p_2$. This yields $\Box p_2 \wedge \neg \Box p_3$
- We try to take all higher alternatives in S and apply MAX_{AL} to them, but the modal has thrown all higher alternatives of S in the bin. We only have $\Box p_0$ left, which is ranked lower than $\Box p_3$, and even if we had higher alternatives, $\Box p_2$ is no longer in the set of alternatives, so the part $p' >_S p$ in the definition of *at least* is vacuous; there is no longer a p to compare p' to
- Thus, we only derive the possibility $\Box p_2 \wedge \neg \Box p_3$. This is where we need a separate scale for MAX_{AL}. If MAX_{AL} used S, it would be unable to apply to the prejacent because there is no prejacent left in S. The fact that MAX_{AL} uses AL enables it to yield (65) even when S has been flattened to contain only $\Box p_0$
- (65) is not inquisitive, so no epistemic implicatures
- Meaning: Malika is required to adopt two or more cats but she is not required to adopt three or more cats. Authoritative reading with a variation inference: she has to adopt at least two cats, and she is free to choose a number of cats to adopt in the $[3\text{-}\infty)$ range

Example with *at most* and *allowed*:

(66) Malika is allowed to adopt at most two cats.

S is now as in (67):

(67) $S = AL = \Diamond p_0 < \Diamond p_1 < \Diamond p_2 < \Diamond p_3 < \Diamond p_4$ ...

Option 1: no flattening

(68) $[\![\text{Malika is allowed to adopt two cats}]\!]^O = \{\Diamond p_2\}$

Adding *at most*:

(69) $\{\Diamond p_2 \wedge \neg\Diamond p_3,$
$\cup\, \{\Diamond p_0 \wedge \neg\Diamond p_1, \Diamond p_1 \wedge \neg\Diamond p_2\}\}$

- $\textsc{max}_{AL}\ \Diamond p_2$ yields the first possibility: $\Diamond p_2 \wedge \neg\Diamond p_3$
- The second possibility is the union of the possibilities $\textsc{max}_{AL}\ \Diamond p_n$ for all numbers lower than 2: $\Diamond p_0$ and $\Diamond p_1$

Epistemic reading: the speaker does not know whether the upper bound is two or some number under two

Option 2: flattening

Prejacent of the modal:

(70) $[\![\text{Malika adopts two cats}]\!]^O = \{p_2\}$

Alternatives:

(71) $[\![\text{Malika adopts two cats}]\!]^A = \{p_0, p_1, p_2, ...\}$

Flattened alternatives:

(72) $[\![\text{Malika adopts two cats}]\!]^A = \cup\{p_0, p_1, p_2, ...\} = \{p_0\}$

Adding the modal yields (73) and (74):

(73) $[\![\text{Malika is allowed to adopt two cats}]\!]^O = \{\Diamond p_2\}$

(74) $[\![\text{Malika is allowed to adopt two cats}]\!]^A = \{\Diamond p_0\}$

Given that S in an ordered version of the CQ and CQ $= [\![\alpha]\!]^A$, we get:

(75) $S = \Diamond p_0$

AL, being independent of $[\![\alpha]\!]^A$, remains unaffected by this change:

(76) $AL = \Diamond p_0 < \Diamond p_1 < \Diamond p_2 < \Diamond p_3 < \Diamond p_4$

After adding *at most*, we get:

(77) $\{\Diamond p_2 \wedge \neg\Diamond p_3\}$

- As above, this is simply $\textsc{max}_{AL}$ applied to the prejacent
- This is possible because while S has been flattened, AL is still as in (67). As for the other alternatives, there is one alternative that is lower than $\Diamond p_2$, namely $\Diamond p_0$, the only alternative we have left. But according to the definition of *at most*, we have to find all $p' <_S p$. p is the prejacent $\Diamond p_2$, but $\Diamond p_2$ has been taken out of CQ and is therefore no longer ordered by S. As a result, we still cannot pick out any alternative, and are left with just the first possibility
- (77) says that Malika is allowed to adopt two cats but she is not allowed to adopt three cats. Thus, it places an upper bound of two on the number of cats Malika is allowed to adopt. There is no epistemic implicature because there is only one possibility. This is the authoritative reading

4.3 Less natural combinations

Epistemic readings

(78) Malika is allowed to adopt at least two cats.

Ordered alternatives:

(52) $S = AL = \Diamond p_0 < \Diamond p_1 < \Diamond p_2 < \Diamond p_3 < \Diamond p_4$ ...

Denotation:

(79) $\{\Diamond p_2 \wedge \neg\Diamond p_3,$
$\cup \{\Diamond p_3 \wedge \neg\Diamond p_4, \Diamond p_4 \wedge \neg\Diamond p_5, ...\}\}$

- First possibility: $\textsc{max}_{AL}\ \Diamond p_2$
- Second possibility: equivalent to:

  (80) $\Diamond p_3 \wedge \exists p'[p' >_S p_3 \wedge \neg\Diamond p']$

So: either Malika is allowed to adopt two cats but no more, or she is allowed to adopt some other number of cats above two, but there is an upper bound to how many cats she is allowed to adopt. Ignorance reading about where the upper bound is, as desired

(81) Malika is required to adopt at most two cats.

Ordered alternatives:

(40) $S = AL = \Box p_0 < \Box p_1 < \Box p_2 < \Box p_3 < \Box p_4$ ...

Denotation:

(82) $\{\Box p_2 \wedge \neg\Box p_3,$
$\cup \{\Box p_0 \wedge \neg\Box p_1, \Box p_1 \wedge \neg\Box p_2\}\}$

- First possibility: $\textsc{max}_{AL}\ \Box p_2$
- Second possibility: equivalent to

  (83) $\Box p_0 \wedge \neg\exists p'[p' <_S p_2 \wedge \Box p']$

So: either Malika has to adopt at least two cats or she as to adopt some minimum number of cats below two. There is ignorance about the lower bound. This is the epistemic reading we wanted to derive

Authoritative readings

Deriving an authoritative reading of (78) involves using a flattened CQ that only contains $\Diamond p_0$, as before. And like before, using AL we only derive $\textsc{max}_{AL}\ \Diamond p_2$, because there are no alternatives p' left in S such that $p' >_S \Diamond p_2$:

(84) $[\![\text{Malika adopts two cats}]\!]^O = \{p_2\}$

Alternatives:

(85) $[\![\text{Malika adopts two cats}]\!]^A = \{p_0, p_1, p_2, ...\}$

Flattened alternatives:

(86) $[\![\text{Malika adopts two cats}]\!]^A = \cup\{p_0, p_1, p_2, ...\} = \{p_0\}$

Adding the modal yields (87) and (88):

(87) $[\![\text{Malika is allowed to adopt two cats}]\!]^O = \{\Diamond p_2\}$

(88) $[\![\text{Malika is allowed to adopt two cats}]\!]^A = \{\Diamond p_0\}$

Given that S in an ordered version of the CQ and CQ $= [\![\alpha]\!]^A$, we get:

(89) $S = \Diamond p_0$

AL, being independent of $[\![\alpha]\!]^A$, remains unaffected by this change:

(90) $AL = \Diamond p_0 < \Diamond p_1 < \Diamond p_2 < \Diamond p_3 < \Diamond p_4$

After adding *at least*, we get:

(91) $\{\Diamond p_2 \wedge \neg\Diamond p_3\}$

This is clearly not a possible reading of (78); it says that Malika is allowed to adopt *at most* two cats. It is equivalent to the authoritative reading of (66), in (77):

(66) Malika is allowed to adopt at most two cats.

(77) $\{\Diamond p_2 \wedge \neg\Diamond p_3\}$

The authoritative reading of (81) with a flattened CQ containing only $\Box p_0$ comes about as follows:

(92) $[\![\text{Malika adopts two cats}]\!]^O = \{p_2\}$

This is not an inquisitive proposition, so $[\![\alpha]\!]^O$ is already a singleton set: there is nothing there for the modal to flatten. But assuming that the numeral is the focused element in the sentence, $[\![\alpha]\!]^A$ contains multiple possibilities:

(93) $[\![\text{Malika adopts two cats}]\!]^A = \{p_0, p_1, p_2, ...\}$

The modal flattens this set:

(94) $[\![\text{Malika adopts two cats}]\!]^A = \{\{w_{[0]}, w_{[1]}, w_{[2]}, ...\}\} = \{p_0\}$

Adding the lexical meaning of the modal yields the ordinary meaning in (95) and the alternatives in (96).

(95) $[\![\text{Malika is required to adopt two cats}]\!]^O = \{\Box p_2\}$

(96) $[\![\text{Malika is required to adopt two cats}]\!]^A = \{\Box p_0\}$

The CQ is equivalent to $[\![\alpha]\!]^A$:

(97) $\text{CQ} = [\![\alpha]\!]^A = \{\Box p_0\}$

The set AL, on the other hand, is independent from $[\![\alpha]\!]^A$ and therefore stays as it is. Thus, we have:

(98) $S = \Box p_0$

(99) $AL = \Box p_0 < \Box p_1 < \Box p_2 < \Box p_3 < \Box p_4$

Adding *at most* yields:

(100) $\{\Box p_2 \wedge \neg\Box p_3\}$

This is not an attested reading of (81). It is a reading that sets a lower bound: Malika must adopt at least two cats but she need not adopt more. This reading is equivalent to the variation reading of (51) in (65):

(51) Malika is required to adopt at least two cats.

(65) $\{\Box p_2 \wedge \neg\Box p_3\}$

So, (91) is equivalent to (77) and (65) is equivalent to (100):

		at least	***at most***
$\Diamond$	authoritative	(91) $\#\{\Diamond p_2 \wedge \neg\Diamond p_3\}$	(77) $\{\Diamond p_2 \wedge \neg\Diamond p_3\}$
	epistemic	(79) $\{\Diamond p_2 \wedge \neg\Diamond p_3,$ $\cup\{\Diamond p_3 \wedge \neg\Diamond p_4, \Diamond p_3 \wedge \neg\Diamond p_4, ...\}\}$	(69) $\{\Diamond p_2 \wedge \neg\Diamond p_3,$ $\cup\{\Diamond p_0 \wedge \neg\Diamond p_1, \Diamond p_1 \wedge \neg\Diamond p_2\}\}$
$\Box$	authoritative	(65) $\{\Box p_2 \wedge \neg\Box p_3\}$	(100) $\#\{\Box p_2 \wedge \neg\Box p_3\}$
	epistemic	(56) $\{\Box p_2 \wedge \neg\Box p_3,$ $\cup\{\Box p_3 \wedge \neg\Box p_4, \Box p_4 \wedge \neg\Box p_5, ...\}\}$	(82) $\{\Box p_2 \wedge \neg\Box p_3,$ $\cup\{\Box p_0 \wedge \neg\Box p_1, \Box p_1 \wedge \neg\Box p_2\}\}$

Table 2: Summary of denotations

In table 3 I have summarised the types of readings this analysis derives for each combination, again with # signifying unattested readings.

		at least	***at most***
$\Diamond$	authoritative	**#UB**	UB
	epistemic	LB, UB ignorance	UB, UB ignorance
$\Box$	authoritative	LB	**#LB**
	epistemic	LB, LB ignorance	UB, LB ignorance

Table 3: Summary of readings

Two kinds of bounds:

- Bound set by lexical item: LB for *at least*, UB for *at most*
- Bound set by modal: LB for $\Box$, UB for $\Diamond$ (epistemic inference)

Natural combinations: bound of epistemic inference corresponds to lexical bound

Less natural combinations: for the epistemic readings, the bound of the inference does not correspond to the lexical bound, but the lexical bound is maintained. However, for the authoritative readings, this bound disappears completely

Claim: the disappearance of the lexical bound is the reason why these readings are blocked

Implementation 1: pragmatic economy constraint à la Buccola and Spector's (2016:165) Pragmatic economy constraint on numerals: a sentence with a numeral n is infelicitous if replacing this numeral by a different numeral m would result in the same meaning. Minimally rephrasing their constraint for our current purposes as in (101) would not work:

(101) **Pragmatic economy constraint** (non-final)
An LF ϕ containing a numeral modifier M is infelicitous if, for some N distinct from M, ϕ is truth-conditionally equivalent to $\phi[M \mapsto N]$

This would rule out both denotations on the second and the fourth row of table 2

Improved version:

(102) **Pragmatic economy constraint**
For lower-bounded and upper-bounded numeral modifiers M, an LF ϕ containing M is only felicitous if ϕ sets the same bound as M.

This constraint correctly rules out (91) and (100) but not (77) and (65). (102) can be viewed as an economy principle: it is not efficient to use an expression with a certain meaning (in particular: a lower bound or upper bound) only to subsequently remove this meaning in the computation.

Implementation 2: blocking mechanism

Nouwen (2010): whenever a marked form and an unmarked form convey the same meaning, the unmarked form is given precedence, and the marked form is blocked from having this meaning. For him: competition is between class B modifiers and bare numerals, with the bare numeral denotations being less marked

This case: (91) is identical to (77) and (100) is identical to (65). (77) and (65) get their meaning in a less convoluted way. In these cases, the bound set by the numeral modifier corresponds to the bound set by the modal. In the case of (91) and (100), the derivation involves a reversal of the bound. Not quite the same as the notion of marked versus unmarked meanings: the difference is that in one case, the derivation involves the rather complex and counterintuitive step of turning a lower bound into an upper bound or vice versa, while the other derivation does not. The simpler derivation is preferred

5 Authoritative readings with *at most* and universal modals

Recall that (103), unlike its equivalent with *allowed* and *at least*, does have an authoritative reading:

(103) Malika is required to adopt at most two cats.

Proposal: this is actually the surface scope reading, with the following structure:

(104) [$\Box$ [Malika adopts [at most [[2 *many*] cats]]]]

The prejacent of *at most*, with focus on the numeral, gives rise to the Rooth-Hamblin alternatives in (105):

(105) $[\![\text{Malika adopts [two]}_F \text{ cats}]\!]^A = \{p_0, p_1, p_2, p_3 ...\}$

As usual, the CQ is derived from this set and is ordered as S in (106), and we have an equivalent AL:

(106) $S = AL = p_0 < p_1 < p_2 < p_3 < p_4$ ...

Adding *at most*, we derive (107) as the meaning of the prejacent of the modal:

(107) $[\![\text{Malika adopts at most two cats}]\!]^O = \{p_2 \wedge \neg p_3, \{w_{[0]}, w_{[1]}\}\}$

This is an inquisitive proposition containing two possibilities: the possibility that Malika adopts exactly two cats and the possibility that she adopts fewer than two cats

We saw above that a modal optionally flattens both the ordinary semantic value and the alternative semantic value of its prejacent. In this case, flattening $[\![\alpha]\!]^A$ will not do much, because there is no operator above the modal that needs to use the alternatives. *At most* has already done this below the modal.

Flattening $[\![\alpha]\!]^O$, on the other hand, does have an effect. As shown in (108), the modal now turns the inquisitive proposition in (107) into a non-inquisitive proposition containing only one possibility:

(108) $\{\Box \cup \{p_2 \wedge \neg p_3, \{w_{[0]}, w_{[1]}\}\}\} = \{\Box\{w_{[0]}, w_{[1]}, w_{[2]}\}\}$

(108) says that in all accessible worlds, Malika adopts between zero and two kittens and no more. This is the authoritative reading of (81).[4]

6 A note on the nature of epistemic inferences

Coppock and Brochhagen (2013), who were the first to use inquisitive semantics in an account of modified numerals, derived meanings of the form in (110) for sentences like (109)

(109) Malika adopted at least two cats.

(110) $\{p_2, p_3, p_4, p_5, ...\}$

Schwarz (2016) pointed out that there is a problem with this way of deriving epistemic inferences: you cannot utter (109) unless you consider it a possibility that Malika adopted exactly two cats. In general, the numeral modified by *at least* or *at most* must always correspond to one of the possibilities the speaker considers

[4]Note that it is also possible, though not necessary, to derive an additional authoritative reading with a universal modal and *at least* this way. For (51), the prejacent of the modal is as in (i).

(36) Malika is required to adopt at least two cats.

(i) $\{p_2 \wedge \neg p_3, p_3\}$

When the modal flattens (i), we get (ii). This says that in all worlds, Malika adopts two or more cats.

(ii) $\{\Box \cup \{p_2 \wedge \neg p_3, p_3\}\} = \{\Box p_2\}$

Thus, the surface scope configuration that is available when modified numerals occur with universal modals allows for the generation of an authoritative reading with *at most*, which we indeed observe. It also enables the calculation of a harmless extra authoritative reading with *at least*.

They could remedy this by saying that the possibilities in the speaker's information state must be equivalent to the possibilities in the proposition. A speaker who utters (109) must then consider all the possibilities in (110) to be potentially true. But now the epistemic reading is too strong

Ciardelli et al. (2016), inspired by Quantity implicature-based accounts such as Büring (2008), Schwarz (2013), and Kennedy (2015b), solve this problem by saying that (109) denotes the possibilities in (111): either Malika adopted exactly two cats or she adopted some number of cats above two.

(111) $\{p_2 \wedge \neg p_3, p_3\}$

Now we can say that the possibilities in the speaker's information state must correspond to the possibilities in the proposition and generate the right implicatures that way. Either the number in the prejacent of the modified numeral is the right number or it is some number higher than that number, but not all higher numbers have to be live possibilities for the speaker. This is also the method used here

Quality implicatures are more difficult to cancel than quantity implicatures, as mentioned by Ciardelli et al. (2016). As can be observed in (112), this is a correct prediction:

(112) a. Malika adopted at least two cats. #In fact, she adopted four.
b. Malika adopted more than two cats. In fact, she adopted four.

Adding the information that Malika adopted four cats implies exact knowledge of the number of cats she adopted, and this is incompatible with the epistemic inference of *at least*, making (112-a) infelicitous. On the other hand, it is fine to add this information to a *more than* sentence like in (112-b), which suggests that *more than* either does not give rise to epistemic inferences or gives rise to weaker, perhaps quantity, implicatures.

7 Conclusion

- We have seen that class B modified numerals give rise to epistemic inferences that optionally disappear in the presence of a modal. Whether they disappear depends on the combination of modal and modified numeral
- Previous analyses fail to capture the right readings for these combinations and derive readings that are not attested
- The present account does derive all correct readings without deriving any non-existent readings using a more careful analysis of the scope facts in conjunction with an optional flattening mechanism in inquisitive semantics

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