# Focussing on Split Scope 

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## 1 Introduction

Split Scope A phenomenon in certain Germanic languages where an object quantifier seems to take scope partly under and partly over some other operator (Rullmann, 1995; Jacobs, 1980, 1991; de Swart, 2000; Zeijlstra, 2004; Abels \& Martí, 2010; Penka, 2011).

Dutch:
(1) Petronella wil geen koopman trouwen.

Petronella wants no merchant marry.
'Petronella does not want to marry a merchant.'
(2) a. Surface scope: what Petronella wants to do is to marry no merchant $\square>\neg \exists$
b. Inverse scope: there is no specific merchant Petronella wants to marry
$\neg \exists>$
c. Split scope: Petronella does not want to marry a merchant
$\neg>\square>\exists$
German (Penka, 2011):
(3) Bei der Prüfung muss kein Professor anwesend sein.

At the exam must no professor present be.
'There does not have to be a professor present at the exam.'
(4) a. Surface scope: It is required that there be no professor present at the exam$\square>\neg \exists$
b. Inverse scope: There is no professor who is required to be present at the exam $\neg \exists>\square$
c. Split scope: It is not required that there be a professor present at the exam $\neg>\square>\exists$

- Penka (2011) claims that split scope is limited to negative indefinites: expressions like kein in German and geen in Dutch
- Other authors (de Swart, 2000; Abels \& Martí, 2010) state that split scope is limited to downward entailing and non-monotone expressions, such as at most in (5)
(5) Marin mag hoogstens vijf boeken lezen.

Marin may at most five books read.
'Marin is allowed to read at most five books.'
(6) a. Surface scope: Marin has permission to do this: to read between zero and five books $\diamond>\neg$ more than 5
b. Inverse scope: The highest number of specific books Marin has been given permission to read is five
$\neg$ more than $5>\diamond$
c. Split scope: Marin is not allowed to read more than five books
$\neg>\diamond>$ more than 5
Main claim of this talk: Scope splitting expressions are focus-sensitive expressions
I propose that the following elements are focus-sensitive:

| Dutch | English |
| :--- | :--- |
| geen |  |
| hoogstens | at most |
| maximaal | maximally |
| minstens | at least |
| minimaal | minimally |
| alleen / slechts / maar | only |

Table 1: Scope splitting expressions in Dutch and English
This new classification entails that:

- Downward entailingness or non-monotonicity is not a necessary condition for being a scope splitting expression, because minstens and at least are scope splitting expressions
- Downward entailingness or non-monotonicity is not a sufficient condition for being a scope splitting expression, because minder dan and fewer/less than are not scope splitting expressions


## Roadmap

Section 2 Scope-splitting expressions are focus sensitive
Section 3 Not focus sensitive? Not a scope splitting expression
Section 4 More evidence for the proposed classification
Section 5 Crosslinguistic evidence
Section 6 Analysis
Section 7 Two notes on the syntax
Section 8 Open issues and future work
Section 9 Conclusion

## 2 Scope-splitting expressions are focus sensitive

### 2.1 Focus-sensitivity

- Beaver and Clark (2003): 'An expression is focus sensitive if its interpretation is dependent on the placement of focus'
(7) Nathan only bought an [apple] ${ }_{F}$.
$\rightarrow$ he didn't buy anything else
(8) Nathan only $[\text { bought }]_{F}$ an apple.
$\rightarrow$ he didn't do anything else with it
- All expressions in table 1 are sensitive to the placement of focus (see also Krifka (1999) and Geurts and Nouwen (2007) for focus-sensitive accounts of modified numerals)
- Krifka (1999):
(9) a. At least $[\text { three }]_{F}$ boys left.
b. At least [three Boys] ${ }_{\mathrm{F}}$ left.
- (9-a) means: the number of boys that left is three or higher
- (9-b) means: the people that left included three boys
- (9-a) but not (9-b) is a felicitous answer to the question 'how many boys left?'
- (9-b) but not (9-a) is a felicitous answer to the question: 'who left?'
- A similar difference cannot be observed if we use more than:
(10) a. More than [three $]_{F}$ boys left.
b. \#More than $[\text { three BOYS }]_{F}$ left.

Insofar as it is felicitous, the focus in (10-b) is contrastive. (10-b) cannot be used to mean 'three boys left, and other people possibly left too'; fewer than can only target the numeral

- For all expressions in table 1, it is possible to focus multiple different constituents, resulting in a different meaning
- At most:
(11) a. Maggie drank at most $[\text { three }]_{F}$ vodkas.
$\rightsquigarrow$ Maggie drank no more than three vodkas (she possibly drank wine too)
b. Maggie drank at most $[\text { three vodkas }]_{\mathrm{F}}$.
$\rightsquigarrow$ All Maggie drank is three vodkas (she didn’t drink anything else)
- (11) has almost the same meaning as (12):
(12) a. Maggie drank only $[\text { three }]_{\mathrm{F}}$ vodkas.
$\rightsquigarrow$ Maggie drank no more than three vodkas (she possibly drank wine too)
b. Maggie drank only $[\text { three vodKas }]_{\mathrm{F}}$.
$\rightsquigarrow$ All Maggie drank is three vodkas (she didn’t drink anything else)
- Another way to see this: continuations
a. Maggie drank $\{$ at most / only $\}$ three ${ }_{F}$ vodkas, and possibly some wine as well.
b. Maggie drank $\{$ at most / only $\}[\text { three vodKas }]_{\mathrm{F}}, \#$ and possibly some wine as well.
- This is in sharp contrast with fewer than:
a. Maggie drank fewer than $[\text { three }]_{F}$ vodkas.
$\rightsquigarrow$ Maggie drank no more than two vodkas (she possibly drank wine too)
b. \#Maggie drank fewer than [three vodKas] $]_{\mathrm{F}}$.
$\nsim$ All Maggie drank is less than three vodkas (she didn't drink anything else)
- Shifting the focus of (15) results in an infelicitous sentence
- If $(15-b)$ can be used at all, it is in a contrastive context where someone else has said, for example, that Maggie drank fewer than three whiskeys
- The same can be said when we use up to:
a. Maggie drank up to $[\text { three }]_{F}$ vodkas.
$\rightsquigarrow$ Maggie drank between and one and three vodkas (she possibly drank wine too)
b. \#Maggie drank up to [three VodKAs] .
$\nsim$ All Maggie drank is between one and three vodkas (she didn't drink anything else)
- Geen behaves like at least, at most and only in this respect:
a. We vonden geen $[\text { bewijs }]_{F}$ voor fraude.

We found GEEN [evidence] $]_{F}$ for fraud.
${ }^{\prime}$ We didn't find any [evidence] ${ }_{F}$ for fraud.' ( $\rightsquigarrow$ but we did find something else; e.g. something that might still point in the direction of fraud despite not being proper evidence)
b. We vonden geen bewijs voor $[\text { fraude }]_{\mathrm{F}}$.

We found geen evidence for $[\text { fraud }]_{F}$.
'We didn't find any evidence for $[f r a u d]_{F} . '(\rightsquigarrow$ but we did find evidence for something else)

- It is possible to use focus to change the flavour of the sentence
- This cannot be done when we use no in English: we cannot shift the focus, and we do not get the same interpretive differences:
(17) a. We found no evidence for fraud.
b. \#We found no [evidence] $]_{F}$ for fraud
w but we did find something else; e.g. something that might still point in the direction of fraud despite not being proper evidence
c. \#We found no evidence for [fraud] ${ }_{F}$ $\nsim$ but we did find evidence for something else
- In sum: all expressions in table 1 can target different words/constituents depending on focus. Fewer than, up to and no cannot do this


### 2.2 Split scope

- We have already seen that geen and hoogstens / maximaal / at most / maximally give rise to split scope readings: ${ }^{1}$
(1) Petronella wil geen koopman trouwen.

Petronella wants no merchant marry.
'Petronella does not want to marry a merchant.'
(2) a. Surface scope: what Petronella wants to do is to marry no merchant

b. Inverse scope: there is no specific merchant Petronella wants to marry $\neg \exists>$
c. Split scope: Petronella does not want to marry a merchant
(5) Marin mag hoogstens vijf boeken lezen.

Marin may at most five books read.
'Marin is allowed to read at most five books.'
(6) a. Surface scope: Marin has permission to do this: to read between zero and five books $\diamond>\neg$ more than 5

[^0]b. Inverse scope: The highest number of specific books Marin has been given permission to read is five
$\neg$ more than $5>\diamond$
c. Split scope: Marin is not allowed to read more than five books
$\neg>\diamond>$ more than 5

- To see the split readings of minstens / minimaal / at least / minimally, let us take a brief detour to a specific way to derive the correct readings: DegP movement (Kennedy, 1997; Heim, 2000)
- Lexical entries (e.g. Kennedy, 2015):

$$
\begin{align*}
& \llbracket \text { at most } \rrbracket=\lambda m_{d} \lambda P_{\langle d, t\rangle} . \operatorname{MAx}\{n \mid P(n)\} \leq m  \tag{18}\\
& \llbracket \text { at least } \rrbracket=\lambda m_{d} \lambda P_{\langle d, t\rangle} . \operatorname{MAx}\{n \mid P(n)\} \geq m \tag{19}
\end{align*}
$$

- Derivation of a split reading with at most:
(5) Marin mag hoogstens vijf boeken lezen.

Marin may at most five books read.
'Marin is allowed to read at most five books.'
(20) a. Surface scope:
$[\diamond[$ At most $5[\lambda d[\operatorname{Marin}[[\operatorname{reads}][[\mathrm{d}$ many $][$ books $]]]]]]$
b. Split scope:
[At most $5[\lambda d[\Delta[\operatorname{Marin}[[$ reads $][[$ d many $][$ books ] ] ] ] ] ] $]$

$$
\begin{equation*}
\llbracket \text { many } \rrbracket=\lambda n_{d} \lambda P_{\langle e, t\rangle} \lambda Q_{\langle e, t\rangle} \cdot \exists x[\# x=n \wedge P(x) \wedge Q(x)] \tag{21}
\end{equation*}
$$

a. Surface scope:
$\diamond[\max \{n \mid \exists x \wedge \# x=n \wedge \operatorname{book}(x) \wedge \operatorname{read}($ Marin,$x)\} \leq 5]$
$(=(6-\mathrm{a}))$
'Marin has permission to do this: to read between zero and five books.'
b. Split scope:
$\max \{n \mid \diamond[\exists x \wedge \# x=n \wedge \operatorname{book}(x) \wedge \operatorname{read}($ Marin, $x)]\} \leq 5$
$(=(6-\mathrm{c}))$
'The maximum number of books Marin is allowed to read is five or less.'

- Derivation of a split reading with at least:
(23) Marin mag minstens vijf boeken lezen.

Marin may at least five books read.
'Marin is allowed to read at least five books.'
a. Surface scope:
$[\diamond[$ At least $5[\lambda d[\operatorname{Marin}[[\operatorname{reads}][[$ d many $][$ books $]]]]]]]$
b. Split scope:
[At least $5[\lambda d[\diamond[$ Marin [ [reads $][[$ d many $][$ books ] ] ] ] ] ] $]$
a. Surface scope:
$\diamond[\max \{n \mid \exists x \wedge \# x=n \wedge \operatorname{book}(x) \wedge \operatorname{read}($ Marin,$x)\} \geq 5]$
'Marin is allowed to do this: to read five or more books'
b. Split scope:
$\max \{n \mid \diamond[\exists x \wedge \# x=n \wedge \operatorname{book}(x) \wedge \operatorname{read}($ Marin, $x)]\} \geq 5$
'The maximum number of books Marin is allowed to read is five or more'

The difference between (25-a) and (25-b): only (25-b) carries the presupposition that there is an upper bound to the number of books Marin is allowed to read

- $(25-\mathrm{b})$ is $(25-\mathrm{a})$ plus a presupposition. That means that in every situation where you can use (25-b), you can also use ( $25-\mathrm{a}$ ). How can we tell that the split reading in ( $25-\mathrm{b}$ ) really exists and is not just a specific use of $(25-\mathrm{a})$ ?
- Answer: (25-a) is not an available reading; at least and its kin (minimally, minstens, minimaal) must outscope the modal
- Argument 1: if we look carefully, we can see that the surface scope reading does not exist

Scenario: Marin is a school child in a school with the following rules. The children in Year 5 are allowed to read as many books as they like during the school year. The children in Year 6, on the other hand, are expected to focus more on subjects such as maths and geography, and they have an upper limit to the number of books they can read at school. The exact upper limit varies from child to child and depends on the child's reading level and the child's grades for other subjects. In addition, new research has just been published that indicates that children who read 20 books a year or more have better vocabulary than those who read fewer than 20 books a year.

| Year 5 | Year 6 |
| :--- | :--- |
| Unlimited books | Upper limit to the number of books <br> (different for every child) |
| 20 books or more: better vocabulary |  |

Table 2: Summary of the scenario

Gijsbrecht and Marin's dad are talking about this new research. Gijsbrecht wonders about Marin's vocabulary and asks the question in (26).

Gijsbrecht: Zit Marin in groep 5 of in groep 6?
Gijsbrecht: Sits Marin in group 5 or in group 6?
'Gijsbrecht: Is Marin in Year 5 or in Year 6?'
Marin is in Year 6; Marin's dad answers:
a. Marin zit in groep 6. Ze mag meer dan 20 boeken lezen.

Marin sits in group 6. She may more than 20 books read.
'Marin is in Year 6. She is allowed to read more than 20 books.'
b. Marin zit in groep 6. Ze mag \{ minstens / minimaal \} 20 boeken lezen.

Marin sits in group 6. She may \{ at least / minimally \} 20 books read.
'Marin is in Year 6. She is allowed to read \{ at least / minimally \} 20 books.'
$\rightarrow$ Year 6: upper limit; both more than and at least can be used

Marin is in Year 5; Marin's dad answers:
a. Marin zit in groep 5. Ze mag meer dan 20 boeken lezen.

Marin sits in group 5. She may more than 20 books read.
'Marin is in Year 5. She is allowed to read more than 20 books.'
b. \#Marin zit in groep 5. Ze mag \{ minstens / minimaal \} 20 boeken lezen.

Marin sits in group 5. She may \{ at least / minimally $\} 20$ books read.
\#'Marin is in Year 5. She is allowed to read \{ at least / minimally \} 20 books.'
$\rightarrow$ Year 5: no upper limit; more than can be used but at least cannot

- Argument 2: when we put at least in a finite clause; an island for QR, and thereby force it to stay under an existential modal, the resulting sentence is infelicitous

Dutch:
(29) a. Het is wettelijk verplicht dat biologische kippen meer dan $1000 \mathrm{~cm}^{2}$ ruimte It is by law required that organic chickens more than $1000 \mathrm{~cm}^{2}$ space hebben. have.
'It is required by law that organic chickens have more than $1000 \mathrm{~cm}^{2}$ of space.'
b. Het is wettelijk verplicht dat biologische kippen $\{$ minstens / minimaal \} 1000 It is by law required that organic chickens $\{$ at least / minimally $\} 1000$ $\mathrm{cm}^{2}$ ruimte hebben.
$\mathrm{cm}^{2}$ space have.
'It is required by law that organic chickens have $\{$ at least / minimally $\} 1000 \mathrm{~cm}^{2}$ of space.'
(30) a. De overheid staat toe dat verplegers meer dan 40 uur per week werken. The government allows that nurses more than 40 hours per week work. 'The government allows nurses to work more than 40 hours a week.'
b. \#De overheid staat toe dat verplegers \{ minstens / minimaal \} 40 uur per week werken. The government allows that nurses $\{$ at least / minimally $\} 40$ hours per week work. 'The government allows nurses to work \{ at least / minimally \} 40 hours a week.'

English:
(31) a. The government requires that organic chickens have more than $1000 \mathrm{~cm}^{2}$ of space.
b. The government requires that organic chickens have \{ at least / minimally \} 1000 $\mathrm{cm}^{2}$ of space.
(32) a. New government regulations allow that nurses work more than 40 hours a week.
b. \#New government regulations allow that nurses work \{ at least / minimally $\} 40$ hours a week.

- Thus: minstens / minimaal / at least / minimally must outscope existential modals
- The split reading, where there is a presupposed upper bound, is the only possible reading
- The inverse scope reading of (23) is the reading that Marin is allowed to read at least five specific books, as in (33). As the split scope reading does not concern specific books, the two readings are distinct. ${ }^{2}$
(23) Marin mag minstens vijf boeken lezen.

Marin may at least five books read.
'Marin is allowed to read at least five books.'

[^1]Inverse scope:
$\max \{n \mid[\exists x \wedge \# x=n \wedge \operatorname{book}(x) \wedge \diamond \operatorname{read}($ Marin, $x)]\} \geq 5$
'The number of specific books Marin is allowed to read is five or higher'

- As the split reading cannot be derived from the surface scope reading or the inverse scope reading, it must be real. So, despite being upward monotone, at least gives rise to split scope readings
- I have shown in this section that whenever an expression is sensitive to focus, it will give rise to split readings, regardless of its monotonicity properties


## 3 Not focus sensitive? Not a scope splitting expression

- As we saw earlier, the expressions minder dan and fewer/less than are not focus sensitive, unlike at most:
(11) a. Maggie drank at most $[\text { three }]_{F}$ vodkas.
$\rightsquigarrow$ Maggie drank no more than three vodkas (she possibly drank wine too)
b. Maggie drank at most [three VodKAs] ${ }_{\mathrm{F}}$.
$\rightsquigarrow$ All Maggie drank is three vodkas (she didn’t drink anything else)
a. Maggie drank fewer than $[\text { three }]_{F}$ vodkas.
$\rightsquigarrow$ Maggie drank no more than two vodkas (she possibly drank wine too)
b. \#Maggie drank fewer than [three vodKas $]_{\mathrm{F}}$.
$\nsim$ All Maggie drank is less than three vodkas (she didn’t drink anything else)
- Minder dan and fewer/less than also do not consistently give rise to split scope readings. The most prominent reading of (34) is a surface scope reading, in sharp contrast with (5) ${ }^{3}$
(34) Marin mag minder dan vijf boeken lezen.

Marin may fewer than five books read.
'Marin is allowed to read fewer than five books.'
(5) Marin mag hoogstens vijf boeken lezen.

Marin may at most five books read.
'Marin is allowed to read at most five books.'

- While (5) sets an upper bound to the number of books Marin is allowed to read, (34) does not, as further evidenced by (35) and (36)
(35) Marin mag minder dan vijf boeken lezen, maar meer mag ook. Marin may fewer than five books read, but more may too. 'Marin is allowed to read fewer than five books, but more is allowed too.'
(36) Marin mag hoogstens vijf boeken lezen, \#maar meer mag ook.

Marin may at most five books read, \#but more may too.
'Marin is allowed to read at most five books, \#but more is allowed too.'

- Another example with an ability modal:

[^2]Dafne Schippers kan in 21.63 seconden \{ hoogstens / maximaal \} 200 meter rennen. Dafne Schippers can in 21.63 seconds \{ at most / maximally \} 200 meters run.
'Dafne Schippers can run \{at most / maximally \} 200 meters in 21.63 seconds.'
a. \#Surface scope reading: It is possible for Dafne Schippers to run between 0 and 200 metres in 21.63 seconds
$\diamond>$ at most
b. Split scope reading : Dafne Schippers cannot run more than 200 metres in 21.63 seconds $\neg>\diamond>$ more than
(39) Dafne Schippers kan in 21.63 seconden minder dan 200 meter rennen.

Dafne Schippers can in 21.63 seconds less than 200 meters run.
'Dafne Schippers can run less than 200 meters in 21.63 seconds.'
(40) a. Surface scope reading: It is possible for Dafne Schippers to run between 0 and 200 metres in 21.63 seconds $\diamond>$ fewer than
b. ??Split scope reading : Dafne Schippers cannot run more than 200 metres in 21.63 seconds
$\neg>\diamond>$ more than

- There are some cases where fewer than does appear to split its scope, which I will get back to. In any case, we can conclude that it is not an across-the-board scope splitting expression like at most
- So, despite being downward monotone, fewer than does not straightforwardly give rise to split scope readings. This is because it is not a focus-sensitive expression


## 4 More evidence for the proposed classification

- All scope splitting expressions must outscope existential modals, as they do in (41):
(41) a. Nigella mag geen taarten bakken.

Nigella may no cakes bake.
'Nigella is not allowed to bake any cakes.'
b. Nigella mag \{ hoogstens / maximaal \} vijf taarten bakken.

Nigella may \{ at most / maximally \} five cakes bake.
'Nigella is allowed to bake $\{$ at most / maximally $\}$ five cakes.'
c. Nigella mag $\{$ minstens / minimaal $\}$ vijf taarten bakken.

Nigella may $\{$ at least / minimally $\}$ five cakes bake.
'Nigella is allowed to bake at $\{$ at least / minimally \} five cakes.'
d. Nigella mag \{ alleen / maar / slechts \} vijf taarten bakken.

Nigella may \{ only / only / only \} (five) cakes bake.
'Nigella is allowed to bake only (five) cakes.'

- We have already seen that this is the case for the expressions like at most and at least ((41-b) and (41-c)):
(5) Marin mag hoogstens vijf boeken lezen.

Marin may at most five books read.
'Marin is allowed to read at most five books.'
(23) Marin mag minstens vijf boeken lezen.

Marin may at least five books read.
'Marin is allowed to read at least five books.'

- For (41-a), the two readings are:
(42) a. Surface scope: Nigella has permission to do this: to not bake any cakes $\diamond>$ no cakes
b. Split scope: Nigella does not have permission to bake any cakes $\neg>\diamond>$ cakes

The surface scope reading does not forbid Nigella to bake cakes; it merely gives her permission not to. This is not a possible reading of (41-a) (Iatridou \& Zeijlstra, 2010)

- Finally, the only sentence in (41-d) does not have the surface reading that it is allowed for Nigella to bake only five cakes. It only has a reading where only restricts what is allowed:
(43) a. Surface scope: Nigella has permission to do this: to bake only five cakes $\diamond>$ only 5 cakes
b. Split scope: Nigella only has permission to bake 5 cakes only $>\diamond>5$ cakes
- Thus, something that all expressions from table 1 have in common is that they all obligatorily outscope existential modals
- As we saw earlier, this is not the case for fewer than or more than
- This indicates that they form a natural class


## 5 Crosslinguistic evidence

- We saw earlier that no in English behaves very differently from geen when it comes to focus:
a. We vonden geen $[\text { bewijs }]_{F}$ voor fraude.

We found GEEN [evidence] ${ }_{F}$ for fraud.
'We didn't find any [evidence] ${ }_{F}$ for fraud.' ( $\rightsquigarrow$ but we did find something else; e.g. something that might still point in the direction of fraud despite not being proper evidence)
b. We vonden geen bewijs voor [fraude] $]_{F}$.

We found GEEN evidence for $[\text { fraud }]_{\mathrm{F}}$.
'We didn't find any evidence for $[\text { fraud }]_{\mathrm{F}}$. ' $(\rightsquigarrow$ but we did find evidence for something else)
a. We found no evidence for fraud.
b. \#We found no [evidence] $]_{F}$ for fraud
$⿻$ \& but we did find something else; e.g. something that might still point in the direction of fraud despite not being proper evidence
c. \#We found no evidence for $[\text { fraud }]_{F}$
$\nsim$ but we did find evidence for something else

- The behaviour of no is not only in contrast with geen but also with not:
(44) a. We didn't find evidence for fraud.
b. We didn't find $[\text { evidence }]_{F}$ for fraud
$\rightsquigarrow$ but we did find something else; e.g. something that might still point in the direction of fraud despite not being proper evidence
c. We didn't find evidence for [fraud $]_{F}$ $\rightsquigarrow$ but we did find evidence for something else
- Thus, no is less free in what it can modify than geen and not. Another way to see this: no cannot modify numbers, unlike geen:

Nigella heeft geen twintig taarten gebakken.
Nigella has GEEN twenty cakes baked.
'Nigella has not baked twenty cakes.'
*Nigella baked no twenty cakes.

- Some cases of split scope with no have been reported in the literature:
(47) The company need fire no employees. (Potts, 2000)
- However, these cases are very rare. For instance, as soon as you change the modal, the split reading disappears:
(48) You have to wear no tie.
$\nsim$ You don't have to wear a tie
- Dutch split scope cases from this handout translated to English:
(1) Petronella wil geen koopman trouwen.

Petronella wants no merchant marry.
'Petronella does not want to marry a merchant.'
(49) ??Petronella wants to marry no merchant.

必'Petronella does not want to marry a merchant.'
(44) a. Nigella mag geen taarten bakken.

Nigella may no cakes bake.
'Nigella is not allowed to bake any cakes.'
??Nigella \{ can / is allowed to \} bake no cakes.
$\nsim$ Nigella is not allowed to bake any cakes

- English no is not focus-sensitive and does not easily give rise to split scope readings like geen
- In addition, no does not have to outscope existential modals. Though (51) is odd, insofar as it has an interpretation, it is a surface scope interpretation. The continuation in (51) seems consistent with the first part:
??Nigella $\{$ can / is allowed to \} bake no cakes, but she can also choose to do some baking if she wants to.

This is another sign that it does not belong in the category of scope splitting expressions

- This is part of a broader pattern for $n$-words in Germanic:
- $n$-words in Dutch, German, Frisian and Icelandic are more free in what they can modify, they can take numbers as arguments, and they give rise to split scope readings easily and consistently
- $n$-words in English, Swedish and Norwegian are less free in what they can modify, they cannot take numbers as arguments, and they do not give rise to split scope readings easily and consistently
- The behaviour of no (and other expressions like it) lends support to the proposed classification: no is not focus-sensitive, does not have to outscope existential modals, and generally does not give rise to split scope readings


## Interim summary

Claim: The class of scope splitting expressions is defined by focus sensitivity rather than downward monotonicity or non-monotonicity

Arguments:

- At least is upward monotone and splits its scope
- Fewer than is downward monotone and does not split its scope
- The class of scope splitting expressions I defined are similar in another way: they must outscope existential modals
- Crosslinguistically, whenever an n-word such as no, geen, or kein is focus-sensitive, it also gives rise to split readings


## 6 Analysis

### 6.1 Deriving split scope readings

- I propose that sentences that give rise to split scope readings have the structure in (52) (this is different from the structure assumed in most of the literature on modified numerals, e.g. Hackl, 2000; Nouwen, 2010; Coppock \& Brochhagen, 2013; Kennedy, 2015)
- Here the scope splitting expression can either take scope under the modal or over the modal. When it takes scope over the modal, split readings arise

- I assume that these scope splitting expressions can take any argument with a type that ends in $t$. For simplicity, I give the variants that take an argument of type $t$ here (see the appendix for the full definitions):

$$
\begin{align*}
& \text { 【only】 }=\lambda C_{\langle t, t\rangle} \cdot \lambda p_{t} . p . \forall p^{\prime} \in C\left[p^{\prime} \rightarrow\left[p^{\prime} \subseteq p\right]\right]  \tag{53}\\
& \text { 【at most } \rrbracket=\lambda C_{\langle\langle, t\rangle} \cdot \lambda p_{t} . \operatorname{MAX}_{\preceq}\left\{p^{\prime} \mid p^{\prime} \wedge p^{\prime} \in C\right\} \preceq p  \tag{54}\\
& \text { 【at least】 }=\lambda C_{\langle t, t\rangle} \cdot \lambda p_{t} . \operatorname{MAX}_{\preceq}\left\{p^{\prime} \mid p^{\prime} \wedge p^{\prime} \in C\right\} \succeq p  \tag{55}\\
& \text { 【geen】 }=\lambda C_{\langle t, t\rangle} \cdot \lambda p_{t} \cdot\left[\exists p^{\prime} \in C \cdot p^{\prime}\right] \cdot \neg p \tag{56}
\end{align*}
$$

Where：
－$\preceq$ and $\succeq$ refer to the ordering of the alternatives（à la Krifka，1999；Geurts \＆Nouwen，2007）
$-\operatorname{MAX}_{\preceq}=\lambda P_{\langle t, t\rangle} \cdot \iota p_{t} \cdot p \in P \wedge \forall p^{\prime}\left[p^{\prime} \in P \rightarrow p^{\prime} \preceq p\right]$
－$\sim \mathrm{C}$ is the set of focus alternatives（Rooth，1985，1992）
－Derivation of a split reading with geen：${ }^{4,5}$
（1）Petronella wil geen $[\text { koopman }]_{\mathrm{F}}$ trouwen． Petronella wants no merchant marry． ＇Petronella does not want to marry a merchant．＇
（57）Split scope：Petronella does not want to marry a merchant $\neg>\square>\exists$
 $\left.\left.\left.\left.\left.\left.\left.[\text { koopman }]_{F}\right][\sim \mathrm{C}]\right]\right]\right]\right]\right]\right]$
（59）$\quad$ PPetronella wil（ 9 ）koopman trouwen $\rrbracket=\square \exists x[\operatorname{koopman}(x) \wedge \operatorname{trouwt}(x)$（Petronella）$]$
（60） $\mathrm{C}=\left\{\text { Petronella wants to marry } X \mid X \in D_{\langle\langle e, t\rangle, t\rangle}\right\}^{6}=\{$ Petronella wants to marry a lawyer，Petronella wants to marry a politician，Petronella wants to marry a police officer， ．．．\}

```
    \llbracketgeen\rrbracket(\llbracketPetronella wil (Э) koopman trouwen\rrbracket) =
    \exists\mp@subsup{p}{}{\prime}\inC. p'.\neg\square\existsx[\operatorname{koopman}(x)\wedge\operatorname{trouwt}(x)(Petronella)]
```

Presupposition：One of the propositions in C is true
Assertion：The proposition 【Petronella wants to marry a merchant】 is false
－Derivation of a split reading with at least，focus on the numeral：
（62）Marin is allowed to read at least［five］${ }_{F}$ books．
（63）Split scope：MAX $>\diamond>5$ books
＇The maximum number of books Marin is allowed to read is five or more＇
（65）$\llbracket$ Marin is allowed to read（ 3 ）five books $\rrbracket=\diamond \exists x[\# x=5 \wedge \operatorname{books}(x) \wedge \operatorname{reads}(x)$（Marin）］
 ［five］$]_{\mathrm{F}}$ ］［books ］］［～C ］］］］］］］］
$\mathrm{C}=\left\{\right.$ Marin is allowed to read $P$ books $\left.\mid P \in D_{\langle e, t\rangle}\right\}$
$=\{\ldots$ ，Marin is allowed to read 4 books，Marin is allowed to read 5 books，Marin is allowed to read 6 books，．．．$\}$

[^3]Ordering of the alternatives：
．．．Marin is allowed to read 4 books $\preceq$ Marin is allowed to read 5 books $\preceq$ Marin is allowed to read 6 books ．．．

【at least』（【Marin is allowed to read（ 3 ） 5 books】）$=$
$\operatorname{MAX}_{\preceq}\left\{p^{\prime} \mid p^{\prime} \wedge p^{\prime} \in C\right\} \succeq[\diamond \exists x[\# x=5 \wedge \operatorname{books}(x) \wedge \operatorname{reads}(x)($ Marin $)]]$
＇The highest ordered alternative $p^{\prime}$ out of the set C that is true is equal to or higher than than the proposition that Marin is allowed to read exactly five books＇
$=$＇The highest number of books Marin has permission to read is 5 or more＇
Presupposition：there is an upper bound to the number of books Marin is allowed to read
－Derivation of a split reading with at most，focus on the noun：
（69）Tom is allowed to drink at most $[\text { two BEERS }]_{\mathrm{F}}$ ．
（70）Split scope：max $>\diamond>2$ beers
＇Tom＇s drinking limit is two beers．＇
 two ］［ beers $]_{\mathrm{F}}$［［ $\left.\left.\left.\left.\left.\left.\left.\sim \mathrm{C}\right]\right]\right]\right]\right]\right]\right]$ ］
（72）$\quad$ Tom is allowed to drink $(\exists)$ two beers $\rrbracket=\diamond \exists x[\# x=5 \wedge \operatorname{beers}(x) \wedge \operatorname{drinks}(x)($ Tom $)]$
$\mathrm{C}=\left\{\right.$ Tom is allowed to drink $\left.X \mid X \in D_{\langle\langle e, t\rangle, t\rangle}\right\}$
$=\{\ldots$ ，Tom is allowed to drink two radlers，Tom is allowed to drink three whiskeys， Tom is allowed to drink a bottle of wine，．．．\}

Ordering of the alternatives：
．．．Tom is allowed to drink two radlers $\preceq$ Tom is allowed to drink two beers $\preceq$ Tom is allowed to drink three whiskeys $\preceq$ Tom is allowed to drink a bottle of wine ．．．

【at most $\rrbracket(\llbracket$ Tom is allowed to drink $(Э) 2$ beers $\rrbracket)=$
$\operatorname{MAX}_{\preceq}\left\{p^{\prime} \mid p^{\prime} \wedge p^{\prime} \in C\right\} \preceq[\diamond \exists x[\# x=2 \wedge \operatorname{beer}(x) \wedge \operatorname{drinks}(x)($ Tom $)]]$
＇The highest ordered alternative $p$＇out of the set C that is true is no higher than the proposition that Tom is allowed to drink exactly two beers＇
$=$＇Two beers is the most Tom can drink．＇
－Thus，by adopting the structure and focus－sensitive semantics of scope splitting expressions given in this section，it is possible to derive split scope readings for all the expressions given in table 1 ， and to derive different readings depending on which element is in focus

## 6．2 Split scope with only

－With these tools in hand，it is possible to see that only（or maar／slechts／alleen in Dutch） behaves in the same way as the other scope splitting expressions：
（76）Cookie Monster can eat only $[\text { cookies }]_{F}$ ．
（77）Split scope：The only thing Cookie Monster can eat is cookies only $>\diamond>$ cookies
 cookies ］［～C ］］］］］］］］
$\llbracket$ Cookie Monster can eat $(\exists)$ cookies $\rrbracket=\diamond \exists x[\operatorname{cookies}(x) \wedge \operatorname{eat}(x)($ Cookie Monster $)]$
$\mathrm{C}=\left\{\right.$ Cookie Monster can eat $\left.X \mid X \in D_{\langle\langle e, t\rangle, t\rangle}\right\}$
$=\{$ Cookie Monster can eat pancakes, Cookie Monster can eat risotto, Cookie Monster can eat salad, ... \}
$\llbracket o n l y \rrbracket(\llbracket$ Cookie Monster can eat $(9)$ cookies $\rrbracket)=$
$\diamond \exists x\left[\operatorname{cookies}(x) \wedge \operatorname{eat}(x)(\right.$ Cookie Monster) $] . \forall p^{\prime} \in C\left[p^{\prime} \rightarrow\left[p^{\prime} \subseteq[\diamond \exists x[\operatorname{cookies}(x) \wedge\right.\right.$ eat ( $x$ )(Cookie Monster) ] ] ] ]

Presupposition: Cookie Monster can eat cookies
Assertion: All the alternatives in C are false unless they entail the proposition $\llbracket$ Cookie Monster can eat cookies】

- We need only to covertly move to a position above the modal and to associate with an element below the modal to generate the correct reading
- The scope mechanism I introduced here can be used to get this reading
- Viewed in this way, only is a scope splitting expression just like geen, at most, etc.


### 6.3 Fewer than

- Earlier I discussed a case where split scope is possible but more difficult: fewer/less than (and minder dan in Dutch):

Marin mag minder dan vijf boeken lezen, maar meer mag ook. Marin may fewer than five books read, but more may too.
'Marin is allowed to read fewer than five books, but more is allowed too.'
Marin mag hoogstens vijf boeken lezen, \#maar meer mag ook.
Marin may at most five books read, \#but more may too.
'Marin is allowed to read at most five books, \#but more is allowed too.'

- However split scope is not always impossible:
(82) At MIT one needs to publish fewer than three books in order to get tenure. (Hackl, 2000)
- I propose that in the cases where split scope readings do arise, it is due to a silent ONLY in the structure
- Thus: fewer than is not a scope splitting expression, but split scope readings sometimes arise because of the covert ONLY operator
- The idea: inserting a silent lexical element comes at a cost, and this is why split scope readings are harder to get in these cases than in the cases where there is an overt scope splitting expression
- Derivation of the split scope reading of (82):

Split scope: MAX $>\square>5$ books
'The maximum number of books you need to publish is three'
[TP [ ONLY ] [TP [ need ] [TP [ you ] [vp [ publish ] [DP [ [ fewer than ] [ [ three ] [ ~C ] ] ] [books ] ] ] ] ] ]
$\llbracket$ You need to publish fewer than three books $\rrbracket=\square \exists x[\# x<3 \wedge \operatorname{books}(x) \wedge \operatorname{publish}(x)($ you $)]$
$\mathrm{C}=\{\ldots$, You need to publish 2 books, you need to publish 3 books, you need to publish 4 books, ... $\}$
$\llbracket$ ONLY $\rrbracket(\llbracket$ you publish fewer than three books $)=$
$[\square \exists x[\# x<3 \wedge \operatorname{books}(x) \wedge \operatorname{publish}(x)($ you $)]] . \forall p^{\prime} \in C\left[p^{\prime} \rightarrow\left[p^{\prime} \subseteq[\square \exists x[\# x<3 \wedge\right.\right.$ books $(x) \wedge \operatorname{publish}(x)($ you $)]$ ] ] ]

Presupposition: In all worlds where you get tenure, there is a number of books you publish, and this number is lower than 3
Assertion: For all numbers from 3, it is not the case that you need to publish that many books to get tenure

- This analysis gives us two ways to obtain a split scope reading, and this explains why split scope is sometimes easy and sometimes more difficult to get


## 7 Two notes on the syntax

### 7.1 What do numeral modifiers modify?

- The traditional structure of a sentence with a modified numeral in the literature on modified numerals (e.g. Hackl, 2000; Nouwen, 2010; Coppock \& Brochhagen, 2013; Kennedy, 2015):

- The structure I have proposed:
(55)

- In the traditional structure, the numeral and the noun do not form a constituent
- In addition, the numeral modifier does not c-command the noun. Therefore, it can never modify it in this structure. We have seen that in reality, it can:
(89) Janet drank at least [five BEERS $]_{F}$. (...And possibly also five whiskeys.)
- The structure argued for here is thus better able to capture the behaviour of numeral modifiers


### 7.2 No inverse scope readings

- Aside from the structure given above, another syntactic assumption I make is that the scope splitting expression can move by itself
- Support for this claim comes from the fact that inverse scope readings (where the whole DP containing the scope splitting expression takes wide scope) seem to be absent for the types of sentences I have discussed here
- The four sentences given below are intuitively false in a scenario that makes the inverse scope reading true and the split scope reading false. This indicates that there is no inverse scope reading

Bij het examen hoeft er geen docent aanwezig te zijn. At the exam must-npi there GEEN teacher present to be. 'There does not have to be a teacher present at the exam.'
(91) a. Inverse scope: for no teacher x is it the case that x must be present at the exam
b. Split scope: it is not necessary for any teacher to be present at the exam

Context where the inverse scope reading is true but the split scope reading is false: The school regulations dictate that at least one teacher has to be present at every exam. It does not matter which teacher goes, as long as there is a teacher. For this particular exam, it has not been determined which teacher will be there. So: there is no particular teacher who has the obligation to be present at the exam, but there does have to be a teacher at the exam.
(92) Vera mag geen film kijken

Vera may no film watch.
'Vera is not allowed to watch a film.'
a. Inverse scope: There is no film x such that Vera is allowed to watch x
b. Split scope: Vera is not allowed to watch a film

Context where the inverse scope reading is true but the split scope reading is false: Vera has to ask permission from her parents before she watches a film. She has not done this yet, so there is no film she has been given permission to watch. But she is in principle allowed to watch films. This means that there is no specific film Vera is allowed to watch, but she is allowed to watch a film.
(94) Felix can read at most five books during his holiday.
(95) a. Inverse scope: The maximum number n such that there is a specific group of books with cardinality $n$ Felix can read during his holiday is five or lower
b. Split scope: The maximum number of books Felix can read during his holiday is five or less

Context where the inverse scope reading is true but the split scope reading is false: Felix only reads on his e-reader. He will take his e-reader with him on holiday and he has downloaded four books to read. He may or may not download more books to read when he has reached his destination. In this case, the maximum number such that there is a specific group of books with that cardinality that Felix can read is five, but it is not the case that the maximum number of books he can read is five or less.
(96) Rosa has to present at most two papers.
(97) a. Inverse scope: The maximum number n such that there is a specific group of papers with cardinality $n$ that Rosa has to present is two or lower
b. Split scope: The maximum number of papers Rosa has to present is two or less

Context where the inverse scope reading is true but the split scope reading is false: Rosa has to present five papers. Two of them have been selected for her, and she can choose the other three herself. In this case, the maximum number such that there is a specific group of papers with that cardinality that Rosa has to present is two, but it is not the case that the maximum number of papers she has to present is two or less.

- This is due to a constraint on movement of the whole DP rather than there being something semantically wrong with these readings. When the DPs in question are base generated above the modals, readings where the whole DP outscopes the modal do arise. The sentences below have a true reading in the context below
(98) Geen docent hoeft aanwezig te zijn bij het examen.
(99) At most two teachers have to be present at the exam.

Context where the entire DP takes scope over the modal but the split reading would be false: The school regulations dictate that at least three teachers have to be present at every exam. It does not matter which teachers go, as long as there are at least three teachers. For this particular exam, it has not been determined which teachers will be there. So: there is no particular teacher or group of one or two teachers who have the obligation to be present at the exam, but there do have to be three teachers at the exam.

- For some other accounts on the market, QR of the whole DP is a prerequisite for split scope. For de Swart (2000), split scope comes about by first moving the whole DP and then applying a type shift. For Abels and Martí (2010), there is QR and then selective deletion:

```
[Geen stropdas [ je hoeft geen stropdas te dragen ]]
```



```
'You don't have to wear a tie'
```

- Given that for some reason or other QR seems to be prohibited in this cases, building split scope on top of it is probably not a good idea
- In my account, the scope splitting expression moves by itself. This is compatible with QR being unavailable
- In sum, two arguments for the proposed syntax are:
- The structure I adopt allows the scope splitting expression to c-command any element in the DP
- The fact that the scope splitting expression moves by itself is compatible with there being no real inverse scope readings


## 8 Open issues and future work

### 8.1 Split scope with nominal quantifiers

- Spit scope also seems possible with nominal quantifiers (Jacobs, 1980), when the special 'hat contour' intonation is used (Büring, 1997):
(101) $\quad[\text { Iedereen }]_{F}$ is geen genie.
$[\text { Everyone }]_{\mathrm{F}}$ is GEEN genius.
'Not everyone is a genius.'
\#'Everyone is not a genius.'
- This reading is not there when a neutral intonation is used:
(102) Iedereen is geen genie.

Everyone is GEEN genius.
\#'Not everyone is a genius.'
'Everyone is not a genius.'

- Only geen does this; e.g. hoogstens does not (infelicitous only with the hat contour intonation):
(103) \#[Iedereen $]_{F}$ zag hoogstens vijf vogels.
[Everyone] $]_{F}$ saw at most five birds.
'Everyone saw at most five birds.'
- This is not possible for English no but it is possible for not:
(104) \#[Everyone $]_{F}$ is no genius.
(105) [Everyone $]_{F}$ didn't go.
- Possible explanation of the contrast between (101) and (102): the scope splitting expression targets everyone because it is in focus. (106-a) is fine because 'not everyone' is meaningful. In (106-b), on the other hand, having the subject be 'maximally everyone' makes the sentence in (102) tautologous: it is always the case that maximally everyone saw five birds, because this is compatible with the entire range from no-one seeing five birds up to everyone seeing five birds.
a. $\quad \neg$ [everyone] is a genius
b. max [everyone] saw five people
- Explanation for the lack of split scope reading in (104): no is neither focus-sensitive nor a scope splitting expression, so the process of no taking wide scope and targeting everyone does not take place
- But: this predicts that split scope should arise in (107), because 'maximally most people' is meaningful, contrary to fact
(107) \#De [meeste] $]_{\mathrm{F}}$ mensen zagen hoogstens vijf vogels.

The most people saw at most five birds.
'Most people saw at most five birds.'

### 8.2 Exactly

- Exactly is a possible counterexample
- It does not seem to be able to target anything other than the numeral it modifies regardless of the placement of focus:
(108) a. Maggie drank exactly [three] ${ }_{F}$ vodkas.
b. \#Maggie drank exactly [three vodKas $]_{\mathrm{F}}$.
$\nprec$ The exact thing Mary drank is: three vodkas
- Exactly does give rise to split readings if you assume it has a maximality operator in its semantics, as in the degree quantifier lexical entry in (109) or the focus-sensitive lexical entry in (110):

$$
\begin{align*}
& \llbracket \text { exactly }=\lambda m_{d} \lambda P_{\langle d, t\rangle} \cdot \operatorname{MAx}\{n \mid P(n)\}=m  \tag{109}\\
& \llbracket \text { exactly } \rrbracket=\lambda C_{\langle\langle\alpha, t\rangle, t\rangle} \cdot \lambda P_{\langle\alpha, t\rangle} \lambda x_{\alpha} \cdot \operatorname{MAX}_{\preceq}\left\{P^{\prime} \mid P^{\prime}\left(x^{\prime}\right) \wedge P^{\prime} \in C\right\}=P(x) \tag{110}
\end{align*}
$$

Marin is allowed to read exactly five books.
a. Surface scope: Nigella has permission to do this: to bake exactly five cakes $\diamond>$ exactly 5 cakes
b. Split scope: The maximum number of cakes Nigella is allowed to bake is five MAX $5 n>\diamond>\mathrm{n}$ cakes

- Possible solutions:
- Maybe it is more difficult for exactly to target other scales besides the numerical scale. For instance, (113-a) makes sense but (113-b) does not.
a. She is at most an associate professor.
b. ?She is exactly an associate professor.

Perhaps it is this aspect of the meaning of exactly that makes it less likely to target other expressions besides the numeral it modifies rather than the fact that it is not focus-sensitive

- Perhaps a silent only operator is responsible for split readings with exactly. But then the prediction is that these readings are more difficult to get. It is not clear if this prediction is borne out.


### 8.3 The focus-sensitivity of geen

- Recall that the split scope reading of (1) is the one in (61) under the present analysis
(1) Petronella wil geen $[\text { koopman }]_{\mathrm{F}}$ trouwen.

Petronella wants no merchant marry.
'Petronella does not want to marry a merchant.'
$\mathrm{C}=\left\{\right.$ Petronella wants to marry $\left.X \mid X \in D_{\langle\langle e, t\rangle, t\rangle}\right\}$
$=\{$ Petronella wants to marry a lawyer, Petronella wants to marry a politician, Petronella wants to marry a police officer, ... $\}$
$\llbracket$ geen $\rrbracket(\llbracket$ Petronella wil $(\exists)$ koopman trouwen $\rrbracket)=$
$\exists p^{\prime} \in C \cdot p^{\prime} . \neg \square \exists x[\operatorname{koopman}(x) \wedge \operatorname{trouwt}(x)($ Petronella $)]$

Presupposition: One of the propositions in C is true
Assertion: The proposition $\llbracket$ Petronella wants to marry a merchant】 is false

- But the presupposition is only there when there is heavy stress on koopman, which is not necessary for the sentence to be felicitous or to have a split reading
- Possible solution: there is always an alternative which is vacuously true. In this case, if there is no stress on koopman, the true alternative can be, for instance: 'Petronella wants to marry no-one'


### 8.4 Degree quantifiers

- This account predicts that fewer than in (114) cannot undergo degree quantifier movement, because that would result in a split reading
(114) You may eat fewer than five biscuits.
- This could be because it is not a degree quantifier. Comparatives like more than and less/fewer than can modify other things besides numbers, suggesting that a degree quantifier analysis may be too narrow:
a. He is more than a man. (... He's a star.)
b. This is less than a house. (...It's more like a shed.)


## 9 Conclusion

I have argued that scope splitting expressions are focus-sensitive expressions. These expressions can move by themselves to take scope over the modal, resulting in split scope readings.

Benefits of this account:

- A unified account of split scope readings of expressions that are generally considered numeral modifiers and $n$-words
- A better classification of numeral modifiers that excludes fewer than and includes at least
- The possibility to insert a silent ONLY operator gives you two kinds of split scope: when a scope splitting expression is used, split scope is easy and universal; when ONLY has to come to the rescue, it is less prevalent
- The account captures the fact that scope splitting expressions can target different elements depending on whether they are in focus
- It also captures the crosslinguistic facts: some Germanic languages have a non focus-sensitive $n$-word that does not easily give rise to split readings, while others have a focus-sensitive $n$-word that does easily give rise to split readings
- It does not require QR , which is compatible with the fact that inverse scope readings are absent (unlike de Swart, 2000; Abels \& Martí, 2010)
- It maintains the lexical integrity of scope splitting expressions (unlike e.g. Penka, 2011, , who literally splits German kein into two pieces)


## Problems:

- Exactly seems to be a counterexample
- Geen does not always seem to interact with focus alternatives
- Degree quantifier movement for fewer than must somehow be excluded


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## 10 Appendix: Lexical entries

- Lexical entries:

$$
\begin{align*}
& \llbracket \text { only } \rrbracket=\lambda C_{\langle\langle\alpha, t\rangle, t\rangle} \cdot \lambda P_{\langle\alpha, t\rangle} \lambda x_{\alpha} . P(x) . \forall P^{\prime} \in C\left[P^{\prime}(x) \rightarrow\left[P^{\prime}(x) \subseteq P(x)\right]\right]  \tag{116}\\
& \llbracket \text { at most } \rrbracket=\lambda C_{\langle\langle\alpha, t\rangle, t\rangle} \cdot \lambda P_{\langle\alpha, t\rangle} \lambda x_{\alpha} . \operatorname{MAX}_{\preceq}\left\{P^{\prime} \mid P^{\prime}(x) \wedge P^{\prime} \in C\right\} \preceq P  \tag{117}\\
& \llbracket \text { at least } \rrbracket=\lambda C_{\langle\langle\alpha, t\rangle, t\rangle} \cdot \lambda P_{\langle\alpha, t\rangle} \lambda x_{\alpha} . \operatorname{MAX}_{\preceq}\left\{P^{\prime} \mid P^{\prime}(x) \wedge P^{\prime} \in C\right\} \succeq P  \tag{118}\\
& \llbracket \text { geen } \rrbracket=\lambda C_{\langle\langle\alpha, t\rangle, t\rangle} \cdot \lambda P_{\langle\alpha, t\rangle} \lambda x_{\alpha} \cdot\left[\exists P^{\prime} \in C \cdot P^{\prime}(x)\right] \cdot \neg P(x) \tag{119}
\end{align*}
$$

Where:

- $\preceq$ and $\succeq$ refer to the ordering of the alternatives (à la Krifka, 1999; Geurts \& Nouwen, 2007)
$-\operatorname{MAX}_{\preceq}=\lambda \mathcal{Q}_{\langle\langle\alpha, t\rangle, t\rangle} \cdot l P_{\langle\alpha, t\rangle} \cdot P \in \mathcal{Q} \wedge \forall P^{\prime}\left[P^{\prime} \in \mathcal{Q} \rightarrow P^{\prime} \preceq P\right]$


[^0]:    ${ }^{1}$ I will discuss the split scope readings of only later, when the necessary machinery has been introduced.

[^1]:    ${ }^{2}$ Specifically, the split reading does not entail the inverse scope reading. Consider the same scenario as the one given above, except now the teacher does not determine exactly which books Marin is allowed to read. Instead, the teacher has given Marin permission to read three specific books: Jip en Janneke, Nijntje, and Pippi Langkous and the maximum number of books she is allowed to read this month is six. This gives Marin the opportunity to choose three other books herself (or more: after all, the three books the teacher mentioned are books Marin has permission to read, not books she has the obligation to read). In this scenario, the split scope reading of (23) is true: the maximum number of books Marin is allowed to read is higher than five. The inverse scope reading is false: the maximum number of specific books Marin has been given permission to read is three, so this number is not five or higher. Therefore, the split reading does not entail the inverse scope reading.

[^2]:    ${ }^{3}$ To check this, I did a small MTurk test. 90 participants were asked whether they got an upper bound reading in sentences like (34) and (5). People got an upper bound more often in the at most case. People were shown a sentence like (34) or (5) and asked whether Marin was allowed to read more than five books. The three possible answers were 'yes', 'no', and 'there is no way to tell'. For the 90 at most responses, the numbers were 4,40 , and 1 respectively. For the 90 fewer than items, these numbers were 4,29 , and 12 respectively.

[^3]:    ${ }^{4}$ I have inserted a covert existential quantifier here for existential closure，but I am not tied to this particular method． Another way would be to use Partee＇s $A$ operator．
    ${ }^{5}$ For simplicity，I am ignoring the fact that Dutch is generally viewed as an SOV language with V2．
    ${ }^{6}$ This set and the other sets of focus alternatives in this section are of course more restricted than this by the $\sim$ operator．

