The semantics and pragmatics of directional numeral modifiers

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1 Introduction

Directional numeral modifiers are expressions that can be used both as directional prepositions and as numeral modifiers.[1] Expressions that have these two functions exist in a wide variety of languages, a small sample of which are given in (1)-(3).[2]

(1) **English:**

a. Mary walked up to the counter.
b. You can fit up to four suitcases in this car.

(2) **Turkish:**

a. Golun sonuna kadar yuruduk.
Lake edge KADAR we walked.
'We walked up to the edge of the lake.'
b. 10 buvala kadar ucretsiz goturebiilirsiniz.
10 luggage KADAR for free you can take.
'You can take up to ten items of luggage for free.'

(3) **Polish:**

a. Jan idzie do sklepu.
Jan goes DO the store.
'Jan goes up to the store.'
b. Dozwolone do 5 sztuk bagaźu.
Allowed DO 5 items of luggage.
'It is allowed to take up to 5 items of luggage.'

Nouwen (2010) proposes a categorisation of numeral modifiers into two classes on the basis of whether or not they obligatorily give rise to ignorance effects.

[1]Generally, these expressions also have a temporal meaning. English is the exception in that it uses the expression *until* rather than *up to* in temporal contexts.

[2]More specifically, they exist in (at least) fourteen different languages. I obtained data on these languages by sending out questionnaires and interviewing informants. For the complete set of data and judgments I refer the reader to the data file available at URL (to be inserted, URL would reveal the identity of the author).

In response to this, Schwarz, Buccola, and Hamilton (2012) point out that the English expression *up to* differs from the other members in its class and should be categorised separately.

The aim of this paper is twofold. The first is to show that not just *up to* but all directional numeral modifiers (henceforth: DNMs) crosslinguistically form a separate class in the domain of numeral modifiers that has properties that other numeral modifiers do not have. While other upper-bounded numeral modifiers such as *at most* set a strong upper bound, the bound of DNMs is defeasible. Furthermore, DNMs also set a lower bound. This lower bound, unlike their upper bound, is strong: it is entailed by the DNM. Expressions such as *at most*, on the other hand, lack a lower bound.

Furthermore, DNMs display the so-called *bottom-of-the-scale effect* (Schwarz et al. note this about *up to*): they cannot refer to the lowest element of the scale they quantify over. Finally, their monotonicity properties differ from those of other upper-bounded numeral modifiers such as *at most* and *less/fewer than*. Whilst non-directional upper-bounded numeral modifiers are straightforwardly downward monotone and license NPIs, the monotonicity properties of directional numeral modifiers are more complex.

The second goal of the present work is to show that an implicature-based account can explain these properties of DNMs as well as generalising Schwarz et al.'s observation of the bottom-of-the-scale effect to all numeral modifiers. I implement this account in different frameworks to demonstrate its theory-independence.

In the following section, I will discuss the two central observations on which I base my analysis: the data concerning the relative strength of the lower bound and the defeasibility of the upper bound of DNMs. In section 3 I discuss the bottom-of-the-scale effect and monotonicity properties of DNMs. In section 4 I provide an analysis to account for these data. In section 5 I implement the account in the framework of degree semantics (in line with Nouwen, 2010, and Schwarz et al., 2012) and that of inquisitive semantics (e.g. Ciardelli, Groenendijk, & Roelofsen, 2012), akin to Coppock and Brochhagen (2013). In section 6 I discuss the interactions between DNMs and modals. Section 7 discusses the behaviour of DNMs in several different kinds of embedded contexts (under negation, in questions, in conditionals, and with evaluative adverbs). Section 8 explores the evaluative meaning component of DNMs and draws a parallel between DNMs and open-scale adjectives. Section 9 concludes.

2 It's all in the bounds

While there is a rich literature on modified numerals (e.g. Hackl, 2000; Geurts & Nouwen, 2007; Büring, 2008; Nouwen, 2010; Cohen & Krifka, 2010, 2014; Kennedy, 2015; Coppock & Brochhagen, 2013), no attention has thus far been paid to the bounds these expressions set. In this section I show that this is an area where DNMs differ fundamentally from their non-directional counterparts.

2.1 Lower bound

The first difference between DNMs and other upper-bounded numeral modifiers is that DNMs entail the existence of a lower bound. Consider (4).

(4) a. At most three students will show up to the lecture, if any.
b. #Up to three students will show up to the lecture, if any.

The cause of the infelicity of (4-b) is the fact that the second part of the utterance, *if any*, contradicts the first part. As *up to three students will show up* entails that at least one student will show up, subsequently denying this fact leads to an incoherent utterance. (4-a) is felicitous because *at most three students show up* is true in a world in which no students show up. In this sentence, the use of *if any* therefore does not lead to a contradiction.

These facts hold for DNMs crosslinguistically, as exemplified in (5) and (6) for German and Italian.

(5) a. Es werden maximal fünf Studierende zum Seminar
There will maximally five students to the seminar
kommen, wenn überhaupt.
come, if at all.
'There will be maximally five students at the seminar, if any.'
b. #Es werden bis zu fünf Studierende zum Seminar kommen,
There will BIS ZU five students to the seminar come,
wenn überhaupt.
if at all.
'There will be up to five students at the seminar, if any.'

(6) a. Ci saranno al massimo cinque studenti al seminario, se
There will be at most five students at the seminar, if
non nessuno.
not none.
'There will be at most five students at the seminar, if any.'
b. #Ci saranno fino a cinque studenti al seminario, se non
There will be FINO A five students at the seminar, if not
nessuno.
none.
'There will be up to five students at the seminar, if any.'

(7) and (8) provide further evidence for the claim that DNMs assert a lower bound.[3]

(7) a. ?I expect to see at most ten people, but maybe no-one will show up.
b. I expect to see up to ten people, but maybe no-one will show up.

(8) a. ?You're allowed to choose at most two presents, but you can also choose not to select any.

[3]Example (8) was provided by a reviewer of *Console* 2015.

b. You're allowed to choose up to two presents, but you can also choose not to select any.

The infelicity of the a-sentences is a result of the use of *but* in these utterances. *But* indicates a contrast, but as *at most ten/two* is compatible with zero, there is no contrast to be found in these examples. *Up to*'s incompatibility with zero causes an opposition in the b-sentences, licensing the use of *but*.[4]

2.2 Upper bound

Another contrast between DNMs and their non-directional counterparts is that the upper bound set by DNMs appears to be weaker than the upper bound of expressions such as *at most* and *less/fewer than*. More specifically, the upper bound can be cancelled. This is illustrated in (9) and (10).

(9) a. #Peter is allowed to choose at most ten presents, perhaps even more.
 b. Peter is allowed to choose up to ten presents, perhaps even more.

(10) a. #Leftovers keep in the refrigerator for at most one week or more.
 b. Leftovers keep in the refrigerator for up to one week or more.[5]

While the a-sentences clearly express a contradiction, the b-sentences are felicitous. As (11)-(14) exemplify, this property holds for DNMs crosslinguistically. Here the judgments on the b-sentences reflect how good they are as continuations of the a-sentences.

(11) **Greek:**
 a. Mehri triada atoma irthan sto parti.
 MEHRI thirty people came to the party.
 b. Ya tin akrivia, pistevo oti itan eki triada dhio atoma.
 For the preciseness I believe that were there thirty two people.
 'In fact, I believe there were thirty-two people there.'

(12) a. To poli triada atoma irthan sto parti.
 At most thirty people came to the party.
 b. ?Ya tin akrivia, pistevo oti itan eki triada dhio atoma.
 For the preciseness I believe that were there thirty two people.
 'In fact, I believe there were thirty-two people there.'

(13) **Russian:**

[4] In fact, if *but* is replaced by a simple conjunction in (8-a), it is still infelicitous. This is because here, the second part of the utterance is obsolete as it is included in the first part. As expected, this is not the case for (8-b).

(i) a. ?You're allowed to choose at most two presents and you can also choose not to select any.
 b. You're allowed to choose up to two presents and you can also choose not to select any.

[5] Source: `http://minimalistbaker.com/best-ever-5-minute-microwave-hummus/`, last consulted 05-04-2017.

a. Do tridcati ljudej prišlo na večerinku.
Do thirty people came on party.
'Up to thirty people showed up at the party.'

b. Na samom dele, ja dumaju čto tam bylo 32 čeloveka.
In fact, I think that there were 32 people.
'In fact, I believe there were thirty-two people there.'

(14) a. Ne bolee tridcati ljudej prišlo na večerinku.
At most thirty people came on party.
'At most thirty people showed up at the party.'

b. ?Na samom dele, ja dumaju čto tam bylo 32 čeloveka.
In fact, I think that there were 32 people.
'In fact, I believe there were thirty-two people there.'

Furthermore, the upper bound of DNMs can be reinforced, unlike other upper-bounded numeral modifiers. (15) and (16) demonstrate this.

(15) a. #You're allowed to choose at most ten presents, but no more than that.

b. You're allowed to choose up to ten presents, but no more than that.

(16) a. #Speeds of at most 120 km/h are allowed, but you're not allowed to drive faster than that.

b. Speeds of up to 120 km/h are allowed, but you're not allowed to drive faster than that.

2.3 Interim conclusion

The data presented in this section show that DNMs for a separate group in the domain of numeral modifiers in that they crosslinguistically differ from other numeral modifiers in a systematic way. In language after language, it can be observed that DNMs have a cancellable upper bound and a non-cancellable lower bound. As far as I know, no other upper-bounded numeral modifiers display these characteristics. In the following section I will discuss two more differences between DNMs and other numeral modifiers that set an upper bound before moving on to my account of these facts.

3 More differences between DNMs and other numeral modifiers

In addition to the strength of the bounds, DNMs differ from other numeral modifiers that set an upper bound in two ways: they display the so-called *bottom-of-the-scale effect* and they are not downward monotone, as evidenced by the fact that they do not license NPIs. These two properties of DNMs were first discussed by Schwarz et al. (2012) for *up to*. In this section I will discuss Schwarz et al.'s data and show that these data hold for DNMs crosslinguistically rather than only for *up to*.

3.1 The bottom-of-the-scale effect

Schwarz et al. (2012) note that *up to*, unlike *at most*, cannot be combined with the bottom element of the scale it quantifies over. (17) and (18) demonstrate this.

(17) a. At most ten people died in the crash.
b. At most one person died in the crash.

(18) a. Up to ten people died in the crash.
b. #Up to one person died in the crash.

They argue that it is not the number *one* but really the bottom-of-the-scale element that is relevant by showing that as the granularity of the scale changes, so does the bottom-of-the-scale element *up to* is incompatible with. If we assume that eggs are sold in cartons of six only, *up to*, but not *at most*, is incompatible with the number six, as (19) and (20) illustrate.

(19) a. He bought at most a dozen eggs.
b. He bought at most half a dozen eggs.

(20) a. He bought up to a dozen eggs.
b. #He bought up to half a dozen eggs.

(19)-(20) exemplify a case where the bottom-of-the-scale element is higher than one. Because the only possible numbers of eggs that can be bought in this context are divisible by six, the relevant scale is [6, 12, 18, 24, ...] rather than [1, 2, 3, 4, ...]. When this number is lower than one, the bottom-of-the-scale effect (henceforth: BOTSE) still manifests itself, as can be observed in (21)-(22), in a situation where cakes are sold per slice. Assuming that there are 12 slices in a cake, the scale that is relevant in this context is $[\frac{1}{12}, \frac{1}{6}, \frac{3}{12}, \frac{1}{3}, ...]$.

(21) a. She bought at most one cake.
b. She bought at most one slice of cake.

(22) a. She bought up to one cake.
b. #She bought up to one slice of cake.

As these examples show, *up to* is perfectly compatible with the number one when this number is not the BOTS element.

The BOTSE does not only hold for the English expression *up to* but is in fact a property of all DNMs I studied. My informants were consistent in confirming that DNMs are incompatible with the lowest number on the relevant scale in their language. A few examples are given in (23)-(24).

(23) **Spanish**:
a. Como mucho una persona murió en el accidente.
At most one person died in the accident.
b. #Hasta una persona murió en el accidente.
HASTA one person died in the accident.

'Up to one person died in the accident.'

(24) **Farsi:**

a. Hadde aksar yek nafar dar tasadof mord.
At most one person in the crash died.
'At most one person died in the crash.'

b. #Ta yek nafar dar tasadof mord.
TA one person in the crash died.
'Up to one person died in the crash.'

3.2 Monotonicity

Schwarz et al. (2012) claim that *up to* is not downward monotone, although this is a property you would expect to see in an upper-bounded numeral modifier. According to the authors, (25-b) straightforwardly follows from (25-a) but this pattern does not hold for the pair in (26).

(25) a. At most three students smoke. $\models$
b. At most three students smoke cigars.

(26) a. Up to three students smoke. $\not\models$
b. Up to three students smoke cigars.

The argument Schwarz et al. present for their claim that (26-b) does not follow from (26-a) is that in a situation where the speaker knows that one, two, or three students smoke and that one or two — but not three — students smoke cigars, (26-a) is 'true and appropriate', but (26-b) is not.

Two things should be teased apart here. Schwarz et al. argue that (26-b) can fail to be true and appropriate in a situation where (26-a) is both true and appropriate. It is important to recognise that truth and appropriateness are two very different notions that should not be thrown on one pile when judging entailments.

To illustrate, in the scenario described above, (25-a) is also true and appropriate while (25-b) is not. Claiming that the number of cigar smokers is 'at most three' when you know that this number is one or two constitutes a clear quantity violation: why use the less informative 'at most three' when you could have used the more informative 'at most two'? The example provided by Schwarz et al. could therefore just as well have been used to argue that *at most* is not downward entailing.

However, it does appear to be true that *up to* is not downward entailing, as evidenced by the fact that it does not license NPIs, unlike *at most.* Under Ladusaw's (1979) theory of NPI-licensing, where there is a correlation between NPI licensing and downward entailingness, an expression that fails to license NPIs is expected to be either non-monotone or upward entailing. Schwarz et al. present the data in (27)-(28) to illustrate that *up to* does not license NPIs.

(27) a. At most three people had ever been in this cave. (Krifka, 2007)

b. At most three students give a damn about Pavarotti.
(Chierchia & McConnell-Ginet, 2000)

(28) a. *Up to three people had ever been in this cave.
b. *Up to three students gave a damn about Pavarotti.

Although Schwarz et al. claim that the pattern in (26) does not hold, my informants did not straightforwardly reject this pattern in their languages. Furthermore, they did not reject the opposite pattern, either, and were thus unable to determine the direction of entailment. They had no trouble confirming Schwarz et al.'s judgments on the entailment pattern of the equivalent of *at most* in their languages. In addition, my informants were uniform in confirming that DNMs do not license NPIs, as is exemplified for Danish and Dutch in (29)-(30).

(29) **Danish** (NPI: *nogensinde*):
a. Højest fem personer har nogensinde været her.
At most five people have ever been here.
b. *Op til fem personer har nogensinde været her.
OP TIL five people have ever been here.
'Up to five people have ever been here.'

(30) **Dutch** (NPI: *hoeven*):
a. Er hoeven maximaal vijf studenten te komen.
There must-NPI maximally five students to come.
'At most five students have to show up.'
b. *Er hoeven tot vijf studenten te komen.
There must-NPI TOT five students to come.
'Up to five students have to show up.'

In the following section, I will propose an account based on two generalisations that explain this curious behaviour of DNMs, as wall as the bottom-of-the-scale effect discussed earlier and the cancellability effects we saw in the previous section.

4 An implicature-based account

4.1 Two generalisations

To account for the contrasts between DNMs on the one hand and other upper-bounded numeral modifiers on the other, I propose the following two generalisations:

1. The lower bound of DNMs is asserted while their upper bound is implicated.

2. All so-called class B numeral modifiers (Nouwen, 2010) require quantification over a range of values.

Let us first consider generalisation 1. This generalisation follows straightforwardly from the observations about the bounds of DNMs I presented in section 2. The fact that the upper bound is defeasible indicates that it is not an entailment but an implicature. The opposite is true for the lower bound: as it cannot be cancelled, it must be part of the truth conditions of DNMs.

Moreover, this generalisation ties in nicely with the meaning DNMs have in directional and temporal contexts. Here, too, there is a contextually determined 'lower bound' or starting point and a defeasible 'upper bound' or end-point.

(31) Joan worked (from 9am) until 10pm today. She may have even stayed later than that.[6]

(32) Harry ran (from school) all the way up to his house. I think he may even have gone on to run to the football field after that.

In these cases, the relevant starting point is always there. It may be explicitly mentioned or left implicit, but it can never be nonexistent. The end-point, however, is defeasible; (31) and (32) show that it can always be surpassed.

The second generalisation is inspired by Schwarz et al. (2012), who claim that the meaning of *up to* entails a range requirement. I argue instead that this range requirement holds for a broader class of numeral modifiers, namely *class B* numeral modifiers. In the next section I will discuss the distinction between class A and class B modifiers proposed by Nouwen (2010) are and provide the data that show that these modifiers must quantify over a range of values. In section 4.3 I will show that DNMs are also class B modifiers and should therefore also be subject to the range requirement. In section 4.4 I will show that these two generalations can also account for the data I presented in section 3.

4.2 Ranges and ignorance

Nouwen makes a distinction between two kinds of numeral modifiers: class A and class B modifiers. Class B modifiers are defined by their inability to refer to precise numbers. Class A modifiers, on the other hand, do have this ability.

As they cannot refer to specific cardinalities, class B numeral modifiers must always quantify over a plurality of elements or a range of admissible or possible options. This is shown in (33).

(33) a. Computers of this kind have at most 2GB of memory.
b. John is allowed to bring at most 10 friends.

In (33-a), *at most* quantifies over a range of memory capacities of different computers, provided by the plural *computers*. In (33-b), the relevant range is supplied by the modal *allowed*. The elements of the range are permissible numbers of friends John brings.

When a sentence has no operator that creates a range, such as a plural, a modal, or a universal quantifier, the modified numeral is interpreted with

[6] As mentioned in footnote 1, the fact that *until* rather than *up to* is used in temporal contexts is an idiosyncrasy of English.

respect to a range of epistemic possibilities considered by the speaker; the so-called *ignorance effect*. This is illustrated in (34).

(34) a. I will be absent for at most two weeks.
b. #A hexagon has at most 10 sides.

The only interpretation of (34-a) is one where the speaker is unsure when exactly she will return to work; she only knows that her absence will last no longer than two weeks. (34-a) excludes precise knowledge of the duration of absence, unlike its equivalent with a class A numeral modifier, given in (35).

(35) I will be absent for less than two weeks.

As demonstrated in (36), it is possible to specify the exact number referred to by a phrase with the class A modifier *less than*, but not with the class B modifier *at most*. This shows once more that there is a clear incompatibility between class B numeral modifiers and precise numbers.

(36) a. I will be absent for less than two weeks; ten days to be precise.
b. #I will be absent for at most two weeks; ten days to be precise.

This also explains the anomaly of (34-b). By using a class B modifier, the speaker suggests that he considers three and eight to be possible numers of sides of a hexagon. Clearly, this is incompatible with the fact that a hexagon has a fixed number of sides. (37) shows that this effect does not occur when a class A modifier is used.

(37) A hexagon has fewer than 10 sides. (...Namely six.)

Thus, numeral modifiers can be defined according to their ability to refer to a specific cardinality. Class A modifiers have this ability, whilst class B modifiers must always quantify over a range of possible values. All languages I studied are sensitive to the A/B distinction: they all have expressions of both kinds.

4.3 Ranges and DNMs

As noted by both Nouwen and Schwarz et al., *up to* is a class B numeral modifier. (38) illustrates this.

(38) a. #A hexagon has up to 10 sides.
b. #There will be up to 100 people at the party, 90 to be precise.

Up to is incompatible with an exact number. In the absence of a modal or a plural, this leads to ignorance effects. When a modal or plural provides a range for *up to* to quantify over, as in (39), the most prominent reading is one where there is no ignorance. These facts are exactly the same as those we saw in the previous section.[7]

[7] In fact, there appears to be a difference between the prominence of ignorance effects of *up to* and *at most*. The sentences in (33) also have an ignorance reading. In this reading,

(39) a. Computers of this kind have up to 2GB of memory.
b. John is allowed to bring up to 10 friends.

Not only *up to* but all DNMs belong in class B. Examples from Romanian and French illustrate this fact in (40)-(41). The a-sentences and b-sentences are included to show that these languages are sensitive to the class A/class B difference. Hence, the fact that DNMs are incompatible with a precise number in these languages can be attributed to the general distinction discussed in the previous section and is not an idiosyncrasy of the lexical item *până la* and of *jusqu'à*.

(40) a. Un triunghi are mai puţin de 11 feţe.
A triangle has fewer than 11 sides.
b. #Un triunghi are cel mult 10 feţe.
A triangle has at most 10 sides.
c. #Un triunghi are până la 10 feţe
A triangle has PÂNĂ LA 10 sides.
'A triangle has up to ten sides.'

(41) a. Un triangle a moins de 11 côtés.
A triangle has fewer than 11 sides.
b. #Un triangle a au plus 10 côtés.
A triangle has at most 10 sides.
c. #Un triangle a jusqu'à 10 côtés.
A triangle has JUSQU'À 10 sides.
'A triangle has up to 10 sides.'

Thus, in terms of needing to quantify over a range of values, DNMs behave exactly like other class B numeral modifiers. This is the rationale behind the second generalisation I presented at the beginning of this chapter. The next section will illustrate why we need both of these generalisations to account for the full set of data.

4.4 Back to the data

As I mentioned earlier, the data on the bounds of DNMs from section 2 need little explanation: the cancellability facts follow straightforwardly from the generalisation that the lower bound of DNMs is asserted while their upper bound is implicated. Now let us consider the data I discussed in section 3.

First, let us remind ourselves of the bottom-of-the-scale effect. The relevant data are repeated in (42)-(43).

(42) a. At most ten people died in the crash.

(33-a) conveys that all computers have a certain amount of memory capacity and the speaker does not know this amount but she knows that it is no higher than 2GB. (33-b) can express that the speaker is unsure about amount of friends John is allowed to invite, but knows that this amount is lower than 11. These readings seem to be less prominent if not absent for the sentences with *up to* in (39). I will leave this issue for further research.

b. At most one person died in the crash.

(43) a. Up to ten people died in the crash.
b. #Up to one person died in the crash.

Here the DNM *up to* cannot modify the bottom-of-the-scale numeral *one*, while the modifier *at most* can. I argue that this phenomenon is the result of an interaction between the two generalisations I presented in the previous section. Let us make the natural assumption that the relevant scale in (17)-(18) consists of the set of whole numbers. Then *at most* in (17-b) quantifies over the scale [0,1]. *At most* sets a lower bound and is thus compatible with zero. *Up to* in (18-b), on the other hand, does set a lower bound. If we assume that this lower bound is 1, the scale *up to* quantifies over is [1]. If we assume it is higher than 1, the scale is empty. In either case, there is no range of numbers for *up to* to quantify over, so the range requirement is violated.

Since all class B numeral modifiers are subject to the range requirement, I predict that the modifiers that set an upper bound all display the bottom-of-the-scale effect. As (44) demonstrates, this prediction is borne out.

(44) #At most 0 people died in the crash.

Here *at most* can only quantify over [0], which violates the range requirement, so the result is infelicitous.

It seems, then, that the range requirement and its interactions with the bounds of numeral modifiers not only explains the bottom-of-the-scale effect of DNMs but can also account for data on other class B modifiers.

Now let us return to the monotonicity facts. The relevant data are given below.

(45) a. At most three students smoke. $\models$
b. At most three students smoke cigars.

(46) a. Up to three students smoke. ?$\models$
b. Up to three students smoke cigars.

As I mentioned earlier, my informants agreed that (45-a) entails (45-b) but felt that there was something to be said for both directions of entailment in (46). They all confirmed that DNMs in their language do not license NPIs.

What can we make of the fact that my informants think that there is something to be said for both the entailment pattern in (46) and the opposite pattern? Here it will prove to be valuable to tease apart the semantics and the pragmatics of the sentences. Schwarz et al. propose a non-monotone semantics for *up to*. According to the present proposal, DNMs assert a lower bound and are thus upward entailing. The valid pattern is (46-b) $\models$ (46-a) However, the pragmatics of DNMs contributes an upper bound to the meaning. This means that there is a pragmatic force interfering with the judgments and leading informants to think that (46-b) may follow from (46-a). Because the semantics and pragmatics each nudge the informants in a different direction, determining which way the entailment goes becomes quite a challenge. This explains my informants' hesitation to reject either pattern. Furthermore, the fact that DNMs are predicted to be upward entailing is compatible with the fact that they crosslinguistically fail to license NPIs.

4.5 Interim conclusion

In this section I have proposed two generalisations to account for the behaviour of DNMs: DNMs assert a lower bound and implicate an upper bound, and all class B modifiers must quantify over a range of possible values. This explains the ignorance effects class B modifiers give rise to, the defeasibility facts of the bounds of DNMs, the bottom-of-the-scale effect, and the monotonicity and NPI licensing properties of DNMs. The next section contains the technical implementation of this theory in two different frameworks.

5 Implementations in different frameworks

In this section I present a semantics of DNMs that incorporates the two generalisations presented above and that accounts for all the data I have discussed. First I will show that the account works in degree semantics; the framework used by Nouwen (2010) and Schwarz et al. (2012). Then I will demonstrate that inquisitive semantics is equally apt to deal with this account. The purpose of this multi-framework setup is to demonstrate that the observations discussed here are general facts that are not dependent on any particular framework.

5.1 Degree semantics

I propose that DNMs denote that a certain degree predicate holds for all numbers on a scale that starts above zero and ends at the point indicated by the numeral that the DNM modifies.[8]

(47) $[\![\text{up to}]\!] = \lambda n \lambda P \forall m \in [s, ..., n] : P(m)$ where $s > 0$ and $s \neq n$.

'The degree predicate P holds for all numbers m on a scale from a contextually determined starting point s to the number n; the numeral modified by the DNM. The starting point is higher than 0 and the scale consists of at least two elements.'

This definition says that the degree predicate holds of all numbers on a scale that ends at n; the first argument of the DNM. It does not contain a maximality operator or express an upper bound in any other way. Hence the possibility of the degree predicate holding for numbers higher than n is left open. The range requirement is reflected in the requirement that the scale consist of more than one element. The starting point of the scale, s, is determined by the context of the utterance. As illustrated in section 3.1, this number can be lower or higher than one.

[8]This semantics draws heavily on the semantics proposed in Nouwen (2008).

To see how this definition works, let us apply it to (18-a), repeated here as (48). I assume with Hackl (2000) and Nouwen (2010) that a sentence with a modified numeral has the structure in (49): *up to* n is a generalised quantifier over degrees that scopes over the rest of the sentence. The degree trace of the bare numeral is an argument of the counting quantifier *many*, defined in (50).

(48) Up to 10 people died in the crash.

(49) [up to 10 [λn [n-many people died in the crash]]]

(50) $[\![\text{many}]\!] = \lambda n \lambda P \lambda Q. \exists x[\#x = n \wedge P(x) \wedge Q(x)]$

This results in the semantics in (51) for (48).

(51) $\forall m \in [1, ..., 10] : \exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$

As it is, the meaning of a modified numeral with a DNM is now equal to the meaning of bare numerals, given (50): *up to 10* asserts *at least 10* and implicates *no more than 10*, just like the bare numeral *10*. This can be remedied, as in Nouwen (2010), by assuming a blocking mechanism that leads to the insertion of a speaker possibility operator. The idea is that when two expressions have the same meaning, the less complex form is preferred over the more complex form. In line with the Maxim of Brevity (Grice, 1975), it is assumed that it is infelicitous to use a marked form to convey an unmarked meaning. Using *up to 10* rather than *10* to convey the meaning of *10* would be a clear instance of a Brevity violation. To rescue the structure, a speaker possibility operator is inserted and the sentence is interpreted with respect to the options the speaker considers possible. This can be observed in (52).

(52) a. [up to 10 [$\Diamond$ [λn [n-many people died in the crash]]]]
b. $\forall m \in [1, ..., 10] : \Diamond \exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$

In sum, (48) means that the speaker considers it possible that one person died, that two people died, ... , and that ten people died. This seems like an accurate representation of its intuitive meaning.

The insertion of a speaker possibility operator combined with the range requirement results in ignorance effects: since there is always a plurality of numbers on the scale, there will always be multiple possibilities in the speaker's mind.

As s cannot be zero, there is an entailed lower bound. As shown in (53), the bottom-of-the-scale effect is accounted for.

(53) a. #Up to one person died in the crash.
b. [up to 1 [$\Diamond$ [λn [n-many people died in the crash]]]]
c. $\forall m \in [1, ..., 1] : \Diamond \exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$
where $s > 0$ and $s \neq n$.

As s is equal to n, the range requirement is clearly violated, which explains the infelicity of (53-a).

The upper-bound implicature is calculated as follows. If the speaker utters (48) while knowing that 11 ore more people died, the Maxim of Quantity is violated. Hence (54-a) implicates the negation of all alternative propositions with scales ending in numbers above 10, given in (54-b). The combination of (54-a) and (54-b) lead to the statement in (54-c). That is, if m is in the range [1, ...,10] but not in the range [1, ..., 11], [1, ... , 12], [1, ... , 13], etc., the numbers 11, 12, 13, etc. are excluded.

(54) a. $\forall m \in [1, ..., 10] : \Diamond\exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$
$\rightsquigarrow$

b. $\neg\forall m \in [1, ..., 11] : \Diamond\exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)] \wedge$
$\neg\forall m \in [1, ..., 12] : \Diamond\exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)] \wedge$
...

c. $\forall m \in [11, ..., \infty) : \neg\Diamond\exists x[\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$

In this section, I have shown that my account of DNMs can be implemented in the degree semantics framework used by Nouwen and Schwarz et al.. In the following section I will show how the theory can be implemented in inquisitive semantics.

5.2 Inquisitive semantics

5.2.1 The framework

The main difference between inquisitive semantics (e.g Ciardelli, Groenendijk, and Roelofsen, 2009; 2012) and classic semantics is that the framework of inquisitive semantics comprises a richer notion of meaning than classic semantics. Rather than merely expressing truth-conditional content, the meaning of a proposition in inquisitive semantics is taken to be a proposal to update the common ground. This can be done by uttering a proposition that consists of a single possibility or by uttering one that comprises multiple possibilities. When a single possibility is expressed, the meaning of the proposition is equivalent to its meaning in classic semantics. A proposition that conveys multiple possibilities, on the other hand, has a richer structure: it is *inquisitive*. Such an inquisitive proposition is taken to be an invitation or a request to the other participants in the conversation to select one of the possibilities.

To make the distinction between inquisitive and non-inquisitive propositions, the semantics needs to be enriched. To formalise this more fine-grained notion of meaning, the denotation of propositions is raised: a proposition is no longer viewed as a set of worlds, but as a set of sets of worlds. Such a set of worlds is called a *possibility*. A set of sets of worlds is called a *proposition*. Thus, a proposition consists of one or more possibilities. In this framework, then, a non-inquisitive proposition is a singleton set consisting of one possibility, whereas an inquisitive proposition contains at least two possibilities. Ciardelli et al. (2009) define the notions of inquisitiveness and informativeness as follows:

(55)
- A proposition φ is *inquisitive* iff $[\![\varphi]\!]$ contains at least two maximal possibilities.
- φ is *informative* iff $\bigcup[\![\varphi]\!] \neq \omega$

Thus, an inquisitive proposition has multiple possibilities, and a proposition is informative if and only if the set of worlds in an informative proposition $\bigcup[\![\varphi]\!]$ is not equal to the set of worlds in the common ground ω. Put differently, an informative proposition is a proposition that is not a tautology.

The definition of inquisitiveness in (55) states that an inquisitive proposition contains at least two *maximal* possibilities. *Maximal possibilities* are taken to be possibilities that are not included in other possibilities. That is, if possibility α consists of a proper subset of the worlds in possibility β, α is a non-maximal possibility. Non-maximal possibilities, though not contributing any inquisitive or informative content to a proposition, can nevertheless contribute to its meaning: they are taken to draw attention to a certain possibility.[9] For instance, *might* φ conveys two possibilities: φ and ω. As ω is one of the possibilities, no worlds are eliminated: the proposition does not express any informative content. Since φ is a proper subset of ω, it is not a maximal possibility. The sentence thus only contains one maximal possibility, so according to the definition in (55) it is not inquisitive. All it does, then, is draw attention to the possibility φ: it is an *attentive* proposition. Ciardelli et al. (2009) define attentivity as in (56).

(56) φ is *attentive* iff $[\![\phi]\!]$ contains a non-maximal possibility

Coppock and Brochhagen (2013) add one more useful notion to the framework which is useful in discussing the semantics of modified numerals: *interactivity*. This notion is defined in (57)

(57) φ is interactive iff $[\![\varphi]\!]$ contains more than one possibility

This means that a proposition is interactive if and only if it is inquisitive or attentive. In the next section, I show how Coppock and Brochhagen use the inquisitive framework with this modification in their account of modified numerals.

5.2.2 Inquisitive semantics in Coppock and Brochhagen (2013)

Coppock and Brochhagen propose a discourse-based account of the superlative modified numerals *at least* and *at most*, which is partly based on Coppock and Beaver's (2011, 2013) analysis of *only*. Although the authors deal with a wide range of data, I focus here on how they account for the ignorance effects of superlative and other class B modifiers. Their semantics for *at least* and *at most*, slightly modified to versions that take numerals and degree predicates as arguments, is given in (58).

[9]The version of inquisitive semantics I am assuming here is InqA (Ciardelli et al., 2009). In InqB (Ciardelli et al., 2012), possibilities are assumed to be downward closed, so there are no non-trivial non-maximal possibilities.

(58) a. $[\![\text{at least}]\!] = \{\lambda n \lambda P. f\{p \in \hat{s} \mid p \geq_s P(n)\} | f \text{ is a choice function}\}$
b. $[\![\text{at most}]\!] = \{\lambda n \lambda P. f\{p \cap \text{MAX}_s P(n) |\ p \leq_s P(n)\} | f \text{ is a choice function}\}$

Here $\hat{s}$ is the question under discussion relative to a state s. A state is a contextually determined partially ordered set of possibilities (i.e. a partially ordered set of sets of worlds) and the question under discussion is the set of possibilities that are ordered in the state — possibilities in the state that are unordered with respect to one another are not part of the question under discussion. P is a degree predicate, n is a degree, and p and p' are possibilities. The definition of MAX is given in (59).

(59) $\text{MAX}_s(p) = \lambda w. \forall p' \in \hat{s} : p'(w) \rightarrow p \geq_s p'$
The possibility p such that no possibilities in the state $\hat{s}$ are stronger than p

Coppock and Brochhagen's definition of *at least* is the set of possibilities in the state $\hat{s}$ that are at least as highly ordered in s as $P(n)$. For instance, if $P(n)$ is (60-a), the state $\hat{s}$ will be the partially ordered set of possibilities in (60-b) and the question under discussion s will be the set of possibilities in (60-c).

(60) a. Claire drank five beers.
b. $\hat{s}$ = {... , <Claire drank 5 beers, Claire drank 4 beers>, <Claire drank 4 beers, Claire drank 3 beers>, <Claire drank 3 beers, Claire drank 2 beers>, ...}
c. s = {Claire drank 0 beers, Claire drank 1 beer, Claire drank 2 beers, Claire drank 3 beers, Claire drank 4 beers, Claire drank 5 beers...}

If we apply *at least* to (60-a), as in (61-a), we get the semantics in (61-b), which denotes the set of possibilities that are at least as highly ordered in $\hat{s}$ as the possibility that Claire drank five beers. These possibilities are given in (61-c).

(61) a. Claire drank at least five beers.
b. $\{p \in \hat{s} \mid p \geq_s (\exists x[\#x = 5 \wedge \text{beer}(x) \wedge \text{drank}(x)(\text{Claire})])\}$
c. {Claire drank 5 beers, Claire drank 6 beers, Claire drank 7 beers, ...}

At most is defined as the set of possibilities, intersected with the strongest possibility in $P(m)$, that are as highly or less highly ordered in s than $P(n)$. For example, (62-a) has the denotation in (62-b). Still assuming a monotone semantics of bare numerals, $P(5)$ denotes the worlds $\{w_5, w_6, w_7, ...\}$, where w_n stands for *the world in which Claire drank exactly n beers.*[10] From these worlds, MAX removes the worlds that are contained in alternatives higher than $P(5)$, as illustrated in (62-c), which results in an upper bound of five.

[10]Of course this is a simplified model where for each n there is only one world in which Claire drank exactly n beers.

(62) a. Claire drank at most five beers.

b. $\{p \cap \textsc{max}_s P(5) \mid p \leq_s P(5)\}$
where $P(5)$ stands for $\exists x[\#x = 5 \wedge \text{beer}(x) \wedge \text{drank}(x)(\text{Claire})]$

c. $= \{p \cap \{w_0, ..., w_5\} \mid p \in \{p_0, ..., p_5\}\}$
$= \{\{w_0, ..., w_5\}, \{w_1, ..., w_5\}, \{w_2, ..., w_5\}, \{w_3, ..., w_5\}, \{w_4, w_5\}, \{w_5\}\}$

If the semantics of *at most* were the exact mirror image of *at least*, without the maximality operator, the semantics would not generate an upper bound. As will become clear in the next section, this is precisely the sort of semantics that is needed for DNMs.

What is crucial about this analysis is that the denotations of *at least* and *at most* are *interactive*: they contain more than one possibility. Coppock and Brochhagen propose that it is this feature that distinguishes them from class A numeral modifiers, which denote a singleton set. As mentioned in the previous section, a proposition that contains multiple possibilities is taken to be an invitation or a request to the hearer to choose a possibility. Coppock and Brochhagen take this notion one step further: they posit that if a speaker utters an interactive proposition, she must not know which of the possibilities in the proposition are true. If she had known this, she would have uttered a non-interactive proposition with only one possibility. Coppock and Brochhagen call this the *Maxim of Interactive Sincerity*, which they define as in (63).

(63) If φ is interactive, then φ is interactive in the speaker's information set

When is something 'interactive in the speaker's information set'? To make this idea more precise, the notion of interactiveness in a state is defined as in (64), while φ *restricted to the information set* k is defined in (65).

(64) Interactiveness in a state
φ is interactive in an information set k iff the information set k restricted to φ, $[\![\phi]\!] \upharpoonright k$, contains more than one possibility

(65) Restriction
If k is an information set (set of possible worlds) and P is an inquisitive-style proposition, then P restricted to k, $P \upharpoonright k$ is:
$P \upharpoonright k = \textsc{pro}\ \{p \mid \exists q \in P : p = k \cap q\}$

Here PRO ensures that its argument is a proposition. To see how restriction works, let us again turn to the semantics of (62-a). The possibilities denoted by this proposition are repeated in (66-a). Let us assume that the speaker's information state k consists of only one possibility, as in (66-b). Then (62-a) restricted to k will be a singleton set, as illustrated in (66-c).

(66) a. $[\![(62\text{-a})]\!] = \{\{w_0, ..., w_5\}, \{w_1, ..., w_5\}, \{w_2, ..., w_5\}, \{w_3, ..., w_5\}, \{w_4, w_5\}, \{w_5\}\}$

b. $k = \{\{w_4, w_5\}\}$

c. $[\![(62\text{-a})]\!] \upharpoonright k$
$= \{\{w_0, ..., w_5\}, \{w_1, ..., w_5\}, \{w_2, ..., w_5\}, \{w_3, ..., w_5\}, \{w_4, w_5\},$

$\{w_5\}\} \upharpoonright \{\{w_4, w_5\}\}$
$= \text{PRO}\ \{p \mid \exists q \in \{\{w_0, ..., w_5\}, \{w_1, ..., w_5\}, \{w_2, ..., w_5\},$
$\{w_3, ..., w_5\}, \{w_4, w_5\}, \{w_5\}\} : p = \{w_4, w_5\} \cap q\}$
$= \{\{w_4, w_5\}\}$

The proposition in (62-a) is interactive, but it is not interactive in the speaker's information set, which violates the Maxim of Interactive Sincerity. To avoid violations of this maxim, a speaker uttering an interactive proposition must always have an interactive information state: the speaker must be entertaining multiple possibilities in her mind. Class A modifiers do not pose this restriction: as they result in non-interactive propositions, the speaker's information set can but does not have to contain a plurality of possibilities. This, according to the authors, accounts for the fact that class B expressions obligatorily come with ignorance effects in the absence an operator like a modal or a plural, whereas class A expressions are compatible with ignorance but do not require it.

In sum, Coppock and Brochhagen's account of ignorance effects hinges on the idea that class B numeral modifiers are always interactive, and that it is infelicitous to raise an issue with multiple possibilities when you have already settled the issue in your mind. In what follows, I will show that the ignorance effects of DNMs as well as their other properties can also be accounted for in this framework.

5.2.3 DNMs in inquisitive semantics

Coppock and Brochhagen use a maximality operator in their semantics for *at most*, rather than a simple mirror image of their semantics of *at least*, to ensure that *at most* asserts an upper bound. As I mentioned above, something like a mirror image of *at least* is what we need to denote the meaning of DNMs. The semantics of DNMs I propose is given in (67).

(67) $[\![\text{up to}]\!] = \{\lambda n \lambda P. f\{P(m) \mid s \leq m \leq n\} \mid f \text{ is a choice function}\}$
where $s > 0$ and $s \neq n$

This definition says that *up to* is the set of possibilities $P(m)$ such that m is between a contextually determined lower bound s and the number modified by *up to*, n. Again, the existence of a range and of a lower bound above zero are required. To illustrate how this semantics works, let us reuse the example from section 5.1, repeated as (68).

(68) Up to ten people died in the crash.

Using the counting quantifier *many* again, the semantics of (68) is as in (69).

(69) $\{f\{\lambda w \exists x[\#x = m \wedge \text{people}(x)(w) \wedge \text{died-in-the-crash}(x)(w)] \mid$
$s \leq m \leq 10\} \mid f \text{ is a choice function}\}$
$= \{\lambda w \exists x[\#x = m \wedge \text{people}(x)(w) \wedge \text{died-in-the-crash}(x)(w)] \mid s \leq m \leq$
$10\}$
$= \{p_1, p_2, p_3, ..., p_{10}\}$

Here and throughout the rest of this paper I use Coppock and Brochhagen's notational convention to let p_1 mean $\{w_n, w_{n+1}, w_{n+2}, ...\infty\}$, where w_n is the world where the degree predicate is true of exactly n.

The informational content of (68) is now equivalent to that of (70).

(70) At least one person died in the crash.

However, the structure of the proposition will enable us to derive the upper-bound implicature. To do this, we can use Coppock and Brochhagen's inquisitive version of an Exhaustification operation, given in (71).

(71) $\textsc{exh}(P,\hat{s}\ ) = \{p - q \mid p \in P \wedge q = \{w \mid \exists q' \in \hat{s}\ [w \in q' \wedge p \not\subseteq q']\}\}$
where P is the proposition and $\hat{s}$ is the question under discussion

This operator removes all the worlds w from the possibilities p in P whenever w is in one of the possibilities q in $\hat{s}$ and is not entailed by p.

This results in the following outcome for (68):

(72) $\mathrm{P} = \{p_1, p_2, ..., p_{10}\}$ $(= \{\{w_1, w_2, w_3, ...\}, \{w_2, w_3, w_4, ...\}, ..., \{w_{10}, w_11, w_12, ...\}\})$
$\hat{s} = \{q'_0, q'_1, q'_2, q'_3, q'_4, ...\}$ $(= \{\{w_0, w_1, w_2, ...\}, \{w_1, w_2, w_3, ...\}, ...\})$

$$
\begin{aligned}
\textsc{exh}(P,\hat{s}\ ) = {} & p_1 - q = p_1 - \{w_2, w_3, w_4, ...\} = \{w_1\} \\
& p_2 - q = p_2 - \{w_3, w_4, w_5, ...\} = \{w_2\} \\
& ... \\
& p_{10} - q = p_{10} - \{w_{10}, w_{11}, w_{12}, ...\} = \{w_{10}\} \\
& = \{\{w_1\}, \{w_2\}, ..., \{w_{10}\}\}
\end{aligned}
$$

As (72) illustrates, Exhaustification removes all worlds above w_{10} from the informational content, resulting in an implicated upper bound of 10. This is due to the interactive structure of (68). If the denotation of the sentence had been a singleton set (i.e. $\{p_1\}$), the application of EXH would have resulted in $\{\{w_1\}\}$. It is thus the complex inner structure of the proposition and not its informational content — the union of the set of possibilities — that enables us to derive the implicature using this operation.

This procedure no longer works if you assume that propositions are *downward closed*, as is now commonly believed in the inquisitive semantics world (Floris Roelofsen, p.c.). Downward closure is the property that if a proposition contains a certain possibility, it also contains all subsets of that possibility. Briefly, the issue is that all worlds in the denotation will be in a possibility where they survive Exhaustification, so no upper bound implicature will be calculated. Although this is an interesting problem that arises in accounting for the behaviour of modified numerals in inquisitive semantics, it does not affect any of the main points I make here. See the appendix for a detailed explanation of the issue.

As can be seen above, statements with *up to* lead to interactive propositions, just like statements with other class B numeral modifiers. This means that ignorance effects come about in the same way: raising ten different possibilities, as the speaker of (68) does, is infelicitous when you know which possibility is the true one. In exactly the same way as it did in the previous section for *at least* and *at most*, the Maxim of Interactive Sincerity accounts for the ignorance effects of DNMs.

There is one instance where this mechanism does not work, and it is in cases where the lowest number on the relevant scale is used. The proposition denoted by (73) is the singleton set $\{p_1\}$, or, after the application of the Exhaustification procedure, $\{\{w_1\}\}$.

(73) #Up to one person died in the crash.

This sentence does not violate the Maxim of Interactive Sincerity, which only states that the information set of the speaker needs to be interactive *if* the proposition is interactive. Coppock and Brochhagen can therefore not account for the cases in (74).

(74) a. #At most 0 people died in the crash.
b. #At most no-one showed up to the reception.

Again, it is clear that a range requirement is needed to rule out these sentences, both for DNMs and for other class B numeral modifiers.

This section demonstrates that my account for DNMs can easily be transposed from classic degree semantics to the inquisitive semantics framework. In fact, the richer notion of meaning that inquisitive semantics encompasses provides a very natural way to implement these ideas. In this framework, there is no need to assume that equivalence in meaning between bare numerals and modified numerals with a DNM causes the latter to be blocked, resulting in the insertion of a speaker possibility operator. Instead, the difference between bare numerals and modified numerals with a DNM lies in the internal structure of the proposition, with the use of DNMs resulting in interactive propositions. The interactivity of propositions with DNMs provides a straightforward way to account for ignorance effects. In the following section, I discuss how the interactions between DNMs and modals can be modelled in this framework.

6 DNMs and modals

6.1 Modals and modified numerals

In section 4.2 I mentioned that class B numeral modifiers give rise to ignorance effects in simple contexts. For instance, the speaker of (75) indicates by using *at least* that she does not know the exact amount of money Marly saved.

(75) Marly saved at least €100.

When a class B modal or universal quantifier is inserted in the sentence, the most salient reading is one without ignorance. Under this reading the range in (75) is not the range of amounts for which the speaker considers it possible that Marly is required to save that amount of money but rather the range of permitted amounts. In (75), the most salient reading is one where the numbers on the range correspond to different individuals: different people saved different amounts of money, but no one saved less than €100.

(76) a. Marly is required to save at least €100.
b. Everyone saved at least €100.

In this section I focus on cases with modals like (76-a). As Nouwen's (2010) account has some issues dealing with these cases (cf. Nouwen, 2010; Kennedy, 2015), I focus on Coppock and Brochhagen's analysis here. In the following section I discuss how these authors account for the optional disappearance of ignorance effects in the modal cases. In section 6.3 I explore how this analysis can be extended to DNMs.

6.2 Modals in Coppock and Brochhagen

Like most authors (Büring, 2008; Schwarz, 2011; Kennedy, 2015), Coppock and Brochhagen assume that for sentences like (76-a), the ignorance reading arises when the modified numeral takes scope over the modal. The non-ignorance (usually called the 'authoritative') reading is derived when the modal takes scope over the modified numeral. Let us first consider the ignorance reading. Coppock and Brochhagen assume the inquisitive denotation of modals given in (77).

(77) a. $[\![\text{may}]\!] = \{\lambda p.\Diamond p\}$
b. $[\![\text{must}]\!] = \{\lambda p.\Box p\}$

As schematised in (78), a sentence like (76-a) with wide scope for the modified numeral then denotes the possibility that Marly is required to save €100, the possibility that she is required to save €101, the possibility that she is required to save €102, etc. In other words: the speaker knows that Marly is required to save a certain minimum amount of money and she knows that the minimum is €100 or higher, but she does not know the exact minimum. This corresponds to the ignorance reading.

(78) $\{\Box p_n | n \geq 100\}$
$= \{\Box p_{100}, \Box p_{101}, \Box p_{102}, \Box p_{103}, ...\}$

In order to derive the authoritative reading, Coppock and Brochhagen need to assume that the modal takes scope over the modified numeral and that an additional operation takes place: Kratzer & Shimoyama's (2002) Existential Closure operation. This operation takes a set of possibilities and turns it into a single possibility. This yields the denotation of (76-a) schematised in (79).

(79) $\{\cup\{p_{100}, p_{101}, p_{102}, p_{103}, ...\}\}$
$= \{\{w_{100}, w_{101}, w_{102}, w_{103}, ...\}\}$

Because of the Existential Closure operation, the denotation is no longer interactive: it is now a singleton set of worlds; a classic proposition. As it is the interactiveness of propositions with class B modifiers that yields the ignorance effect in Coppock and Brochhagen's account — through the Maxim of Interactive Sincerity — the fact that (79) is not interactive means that there is no ignorance reading. Instead, the meaning is simply that what is required is that Marly saves €100 or more. This is the authoritative reading.

6.3 Modals and DNMs

Coppock and Brochhagen's account can be extended quite straightforwardly to the case of DNMs. Let us again begin with the ignorance reading. For (80), this reading is that there is some maximum number of pieces of luggage Tony can bring and the speaker does not know what this maximum number is, but she knows that it is three or lower.[11,12]

(80) Tony is allowed to bring up to three pieces of luggage.

If the modified numeral takes scope above the modal, the semantics of (80) is as in (81), assuming a lower bound of one.

(81) $\{\lambda w \Diamond \exists x[\#x = m \wedge \text{pieces-of-luggage}(x)(w) \wedge \text{brings}(\text{Tony}, x)(w)] \mid 1 \leq m \leq 3\}$
$= \{\Diamond p_1, \Diamond p_2, \Diamond p_3\}$, where $p_n = \{w_n, w_{n+1}, w_{n+2}, w_{n+3}, ...\}$

Applying Coppock and Brochhagen's Exhaustification procedure then gives us the reading with the upper bound, exactly as above. This reading is schematised in (82).

(82) $\{\Diamond\{w_1\}, \Diamond\{w_2\}, \Diamond\{w_3\}\}$

The meaning that is derived is the ignorance reading: the speaker considers it possible that Tony is allowed to bring one piece of luggage, that he is allowed to bring two pieces of luggage, and that he is allowed to bring three pieces of luggage.

The authoritative reading is derived when the modal takes wide scope. First we calculate the truth conditions without the modal, given in (83).

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[11] As already mentioned in footnote 7, this reading seems to be more difficult to get for DNMs than for other class B modifiers.

[12] In sentences with existential root modals, the bottom-of-the-scale effect seems to disappear, as demonstrated by the felicity of (ii).

(i) Tony is allowed to bring up to one piece of luggage.

However, this is not the case for existential epistemic modals, as illustrated in (ii).

(ii) #Tony may have brought up to one pieces of luggage.

This suggests that we are dealing with a difference between epistemic modals and root modals that is not yet properly understood.

(83) $\{\lambda w \exists x[\#x = m \wedge \text{pieces-of-luggage}(x)(w) \wedge \text{brings}(\text{Tony}, x)(w)] \mid 1 \leq m \leq 3\}$
$= \{p_1, p_2, p_3\}$, where $p_n = \{w_n, w_{n+1}, w_{n+2}, w_{n+3}, ...\}$

Applying the Exhaustification procedure gives us the reading with the upper bound implicature, schematised in (84).

(84) $\{\{w_1\}, \{w_2\}, \{w_3\}\}$

Adding the modal and applying Existential Closure gives us (85).[13]

(85) $\{\Diamond \cup \{w_1\}, \{w_2\}, \{w_3\}\}$
$= \Diamond\{w_1, w_2, w_3\}$

This corresponds to the authoritative reading of (80): it is allowed to bring exactly one piece of luggage, exactly two pieces of luggage, or exactly three pieces of luggage, and no more than that.

Thus, Coppock and Brochhagen's way of deriving the ignorance and authoritative readings with modals can easily be applied to DNMs. In the next section, I explore the behaviour of DNMs in embedded environments.

7 Embedded environments

It is well known that implicatures disappear when they occur in the scope of negation, or, more generally, in downward entailing contexts (Gazdar, 1979; Horn, 1989). (86) illustrates this phenomenon.

(86) a. Leila ordered a cup of coffee or a cup of tea.
b. Leila didn't order a cup of coffee or a cup of tea.

While (86-a) implicates that Leila did not order both a cup of coffee and a cup of tea, (86-b) does not give rise to such an implicature. (86-b) does not implicate that Leila either didn't order a cup of coffee or that she didn't order a cup of tea. Rather, it conveys that Leila ordered neither coffee or tea, meaning that what is negated is the inclusive and not the exclusive reading of the disjunction.

[13] Another possibility is to merge in the modal first and then the exhaustifier. The derivation would then first yield (i) and, after Exhaustification, (ii).

(i) $\{\Diamond p_1, \Diamond p_2, \Diamond p_3\}$

(ii) $\{\Diamond\{w_1\}, \Diamond\{w_2\}, \Diamond\{w_3\}\}$

Existential Closure would then yield (iii), as before.

(iii) $\Diamond\{w_1, w_2, w_3\}$

What is essential is that Existential Closure apply after Exhaustification. Existential Closure turns a set of possibilities into a set of worlds. Because of the meaning of DNMs, this set always includes the bottom-of-the-scale numeral s. Let's say this s is one here. Then Exhaustification will remove all the worlds above the one-world, and this will always yield the incorrect meaning that the only thing that is allowed (or required) is the 'exactly one'-option.

If the upper bound set by DNMs is indeed implicated, the prediction is that, like the exclusive reading of disjunctions, it disappears in downward entailing contexts. In this section, I test this prediction. As I show in section 7.1 below, this prediction is straightforwardly borne out when DNMs occur in the scope of negation. In section 7.2 I discuss the behaviour of DNMs in questions. The picture is more complicated here, but as I will show, this does not seem to be a complication that is specific to DNMs. In section 7.3 I explore interactions between DNMs and evaluative adverbs. The picture that emerges there corresponds exactly what this account predicts.

7.1 Negation

An important first thing to note when discussing DNMs under negation is that they seem to be infelicitous in the direct scope of negation, as illustrated in (87).

(87) #Mary didn't buy up to ten books.

However, they are acceptable when there is a CP that intervenes between the negation and the DNM, as in (88). That is, DNMs appear to be *local PPIs*, (Spector 2014). For this reason, I will restrict my attention to sentences with the structure $[_{\text{TP}} \neg [_{\text{CP}}\ \text{DNM}\ ]\ ]$.

(88) I don't think there will be discounts of up to 70%.

Now let us turn to what this sentence means. As predicted, there is a stark contrast between the meaning of (88) and its counterpart with *at most*, given in (89).

(89) I don't think there will be discounts of at most 70%.

While (88) conveys that the highest discount is *lower* than 70%, (89) means that the highest discount is *higher* than 70%. This intuition was confirmed by my informants: 10 out of 12 speakers observed the aforementioned contrast between (88) and (89) in their language.[14]

This is entirely expected in the current account. Negating a sentence with a DNM means negating the proposition that the degree predicate in the sentence holds for all numbers on the relevant interval. To see what is going on in the case of (88), let us assume that the minimum discount (the bottom-of-the-scale numeral) is 1% and that the scale contains all natural numbers.[15] What is negated is the proposition that for all numbers from 1 to 70, the speaker considers it possible that there will be discounts of that amount. This means there has to be a number in this interval that is not a possible discount according to the speaker. If a monotone semantics of bare numerals is assumed, once the predicate does not hold for a certain number n on the scale, it does not hold

[14] These were speakers of Dutch, French, German, Greek, Italian, Romanian, Russian, Spanish, and Turkish.

[15] Choosing a different bottom-of-the-scale numeral or a different granularity level (e.g. 5%-10%-15%-etc.) does not affect the validity of the reasoning presented here.

for all numbers above n, either. For instance, if 65% is not a possibility, neither are 66%-100%. Hence, by stating that not all numbers in the interval [1-70] are possible discount amounts, the speaker minimally expresses that discounts of 70% or higher are not possible. From this it follows that the highest discount considered is lower than 70%.

At most, on the other hand, expresses a maximum. (89) thus conveys that the maximal discount is 70%. Negating this maximum is equivalent to stating that higher numbers are among the possibilities. The prejacent of (89) expresses *I think the highest discount will be 70%*. Negating this results in the statement that the speaker thinks the highest discount will be higher than 70%.

If an implicature were to arise in (88), the sentence would have the same meaning as (89). The upper bound would be negated, which would result in the proposition that the speaker thinks the discounts are higher than 70%. This is not what the sentence conveys, which suggests that no implicature is calculated in the scope of negation, as predicted.

7.2 Questions

We have seen that there is a contrast between the strength of the upper bound of *at most* and DNMs when they are embedded under negation. The same contrast can be observed in other downward entailing environments. Questions are one environment where this difference manifests itself. I asked eight of my informants to judge the dialogues in (90) and (91) in their language.[16]

(90) Situation: There are 30 rooms in the castle.

Sarah asks: 'Are there at most 20 rooms in the castle?'

Phil answers either A or B:

A. No, there are 30 rooms in the castle.
B. Yes, in fact, there are 30 rooms in the castle.

(91) Situation: There are 30 rooms in the castle.

Sarah asks: 'Are there up to 20 rooms in the castle?'

Phil answers either A or B:

A. No, there are 30 rooms in the castle.
B. Yes, in fact, there are 30 rooms in the castle.

The specific question I asked them was which of Phil's answers would be appropriate answers to Sarah's question: A, B, both, or neither. If A is judged as an appropriate answer, this signifies that the question has an upper-bound reading. If the question is whether 20 is the upper bound and the actual number

[16]These were speakers of Dutch, Farsi, French, German, Greek, Hebrew, and Romanian.

is 30, then it is appropriate to answer 'no' to the question. If B is judged as an appropriate answer, this means that the question has a reading without an upper bound. If the enquiry is about the number 20 and the answer is 'yes', this signifies that the question did not entail an upper bound. Therefore, the prediction was that answer A would be appropriate for the dialogue containing a question with *at most* in (90), and answer B would be appropriate for the dialogue with the *up to* question (91).

This prediction was borne out. Seven out of eight speakers thought B was an appropriate answer in (91), while none of them found B an appropriate answer in (90). It is clear that my speakers observed a large contrast between the strength of the upper bound of *at most* in (90) and *up to* in (91).

Nevertheless, one speaker said that A was the only appropriate answer for (91), and three others thought A and B were both appropriate answers. Thus, even though my informants got the expected contrast, it was still possible for some of them to get an upper bound reading for a DNM in a downward entailing question environment. However, it seems that this phenomenon is not unique to DNMs. I also presented my informants with the dialogue in (92).

(92) Situation: Natalie owns DVDs of all *Star Wars* films.

Daniel asks: Does Natalie own DVDs of some *Star Wars* films?

Ben gives one of the following answers:

A. No, Natalie owns DVDs of all *Star Wars* films.
B. Yes, in fact, Natalie owns DVDs of all Star Wars films.

Some is a textbook case of an expression that gives rise to a scalar implicature. Since *some* is embedded in a question in (92), the prediction is that there is no implicature. *Some* should mean *some and possibly all*, and so the only appropriate answer should be B. Answering A would mean negating *some but not all* and then asserting *all*, which is clearly contradictory. While all of my informants thought that B was a felicitous answer in (92), half of them (four out of eight) thought the A-answer was also possible.

This means that the fact that some of my informants got an upper bound reading for a DNM in a question environment is not evidence against the idea that the upper bound is an implicature but rather seems to challenge the broader claim that implicatures disappear in questions.

One way to account for the presence of implicatures in a downward entailing context such as a question would be to assume the presence of local implicatures, as proposed by Chierchia, Fox, and Spector (2009, 2012) and Spector (2014). In Chierchia et al. (2012), it is observed that the sentences in (93) are coherent only if an implicature is calculated in the scope of negation.

(93) a. Joe didn't see Mary or Sue; he saw both.
b. It is not just that you *can* write a reply. You must.
c. I dont expect that some students will do well, I expect that all

students will.

Without an implicature, all of these sentences are contradictory. For instance, without an implicature the antecedent of (93-a) conveys that Joe did not see Mary, Sue, or both Mary and Sue, while its consequent expresses that he saw both Mary and Sue.

According to the authors, implicatures also seem to appear in other downward entailing contexts such as antecedents of conditionals, as can be observed in (94).

(94) a. If you take salad or dessert, you'll be real full
b. If you take salad or dessert, you pay \$20; but if you take both there is a surcharge.

While there is no implicature in (94-a) — clearly, taking both salad and dessert will also make one feel full — (94-b) can only be interpreted if the disjunction in the antecedent gives rise to a local implicature. That is, the disjunction in the antecedent has to be interpreted as an exclusive disjunction to arrive at a coherent interpretation; without the implicature the sentence asserts that if you take both salad and dessert, you pay \$20 but there is also a surcharge.

If Chierchia et al. are on the right track, this could explain the question data presented above. Both the upper bound in the DNM case and the 'not all' implicature in the *some* case could be instances of local implicatures.

A second possible way to account for the fact that an upper bound implicature might arise in a downward entailing environment is to posit that it is due to the fact that the meaning of a modified numeral containing a DNM is essentially vacuous without the implicature.[17]

Without the implicature, the modified numeral containing the DNM is essentially vacuous. That is, (95-a) would mean almost the same thing as (95-b).

(95) a. Are there up to 20 rooms in the castle?
b. Are there rooms in the castle?

The implicature may still arise in some cases because the modified numeral would not have any contribution to the meaning of these utterances without it. In some cases, the requirement to not use expressions that do not contribute to the meaning of a proposition may force a language user to calculate an implicature in a DE context to arrive at a coherent interpretation. Something comparable may be going on in the *some* case, where (96-a) is very similar in meaning to (96-b).

(96) a. Does Natalie own DVDs of some *Star Wars* films?
b. Does Natalie own DVDs of *Star Wars* films?

Whatever the correct explanation is, the fact that an upper bound implicature for DNMs can arise in questions is not an argument against an implicature

[17] This idea was suggested to me by Spector (p.c.).

analysis of the upper bound as the same facts hold for the 'not all' implicature of *some*.

7.3 Evalutative adverbs

The final test case for the analysis proposed here that I will discuss concerns interactions between DNMs and evaluative adverbs. To see how this test works, consider the sentences in (97).

(97) a. Fortunately, some students attended the wedding. (Nouwen, 2006)
b. Fortunately, the soup is warm.

It is standardly assumed that the part of these sentences without *fortunately* assert respectively that at least some students attended the wedding and that the soup is at least warm. They implicate that not all students attended the wedding and that the soup is not hot. As remarked by Nouwen (2006), *fortunately* targets only the assertion of these sentences, not the implicature. That is, (97-a) cannot be taken to mean that the speaker is happy that not all students attended the wedding and (97-b) conveys that the speaker is glad that the soup is at least warm; she is not necessarily glad that the soup is not hot.

Nouwen uses this observation to explain the difference between *almost* and *not quite*. Both *almost* and *not quite* have two meaning components: a proximal component and a negative component. As (98) illustrates, *fortunately* targets the proximal component in the case of *almost* and the negative component in the case of *not quite*.

(98) a. Fortunately, almost all my friends attended my wedding.
b. Fortunately, not quite all my friends attended my wedding.

(98-a) expresses that the speaker is happy that the number of friends that showed up to his wedding is close to his total number of friends; *fortunately* targets the proximal component of *almost*. In (98-b), on the other hand, the speaker conveys that he is pleased that some friends did not attend the wedding; *fortunately* targets the negative component of *not quite*.

A similar contrast can be observed if one compares *up to* to *at most*. Consider (99).

(99) a. Fortunately, up to 100 people will attend my wedding.
b. Fortunately, at most 100 people attended my wedding.

While the speaker of (99-a) expresses her joy about the high number of guests that will attend the wedding, the person uttering (99-b) conveys that she is happy that no more than 100 people will be there.[18] In parallel to *almost* and *not quite*, *fortunately* targets the positive component of *up to* and the negative component of *at most*.

[18]In fact, DNMs seem to also have a proximal component in some cases, see section 8 for a brief discussion of this fact.

This contrast between DNMs and other upper-bounded numeral modifiers holds crosslinguistically. All of my informants agreed that evaluative adverbs like *fortunately* target the positive component of DNMs and the negative component of *at most* in their languages. (100)-(103) provide an illustration of this fact for Italian and Hebrew.

(100) a. Fortunatamente, posso prendere fino a cinque giorni di ferie.
Fortunately, I can take FINO A five days of time off.
'Fortunately, I can get up to five days off.'

b. ?Fortunatamente, quel pessimo cantante canterà fino a cinque canzoni.
Fortunately, that bad singer will sing FINO A five songs.
'Fortunately, that bad singer will sing up to five songs.'

(101) a. ?Fortunatamente, posso prendere al massimo cinque giorni di ferie.
Fortunately, I can take at most five days of time off.
'Fortunately, I can get at most five days off.'

b. Fortunatamente, quel pessimo cantante canterà al massimo cinque canzoni.
Fortunately, that bad singer will sing at most five songs.

(102) a. Lemarbe ha-simxa / le-mazal-i, ani yaxol le-kabel ad xamisha yemey xofesh.
To much the joy / to luck my, I can receive AD five days of vacation.
'Fortunately, I can getup to five days off.'

b. ?Lemarbe ha-simxa / le-mazal-i, ha-zamar ha-nora'i yashir ad xamisha shirim.
To much the joy / to luck my, the singer the terrible will sing AD five songs.
'Fortunately, that terrible singer will sing up to five songs'

(103) a. ?Lemarbe ha-simxa / le-mazal-i, ani yaxol le-kabel lexol ha-yoter xamisha yemey xofesh.
To much the joy / to luck my, I can receive at most five days of vacation.
'Fortunately, I can get at most five days off.'

b. Lemarbe ha-simxa / le-mazal-i, ha-zamar ha-nora'i yashir lexol ha-yoter xamisha shirim.
To much the joy / to luck my, the singer the terrible will sing at most five songs.

'Fortunately, that terrible singer will sing at most five songs.'

I asked my informants to judge these sentences under the assumption that getting time off work is pleasant while having to listen to the performance of a bad singer is not. The discrepancies in (100-b), (101-a), (102-b), and (103-b) show that in these languages, too, evaluative adverbs target the positive component of DNMs and the negative component of other upper-bounded numeral modifiers.

These facts can only be explained if one assumes that the upper bound of DNMs is not part of their asserted content, as the present analysis predicts.

7.4 Interim conclusion

The data presented in this section provide further evidence for an implicature analysis of the upper bound of DNMs. DNMs behave exactly as predicted under negation and when they occur with evaluative adverbs. When they occur in questions, there is a large contrast between the strength of their upper bound and the strength of the upper bound in the same question with *at most*. Insofar as the implicature survives in these environments, it also does for the classic quantity implicature of *some*.

8 Evaluativity

Besides asserting a lower bound and implicating an upper bound, DNMs also have an evaluative meaning component. When the speaker uses the expression *up to n*, she suggests that *n* is a high number in the context. For instance, the speaker of (104-a) considers 100 to be a high number of guests. The speaker of (104-b) does not express this belief.[19]

(104) a. There will be up to 100 people at my birthday party.
b. There will be at most 100 people at my birthday party.

This is further corroborated by the data in (105). When the speaker explicitly states that she considers the relevant number to be low, it is odd to use *up to* but fine to use *at most*. The reason for the infelicity of (105-a) is that the evaluative meaning component of *up to* contradicts the statement that there were not many visitors.

(105) a. The Louvre didn't get that many visitors: #up to 12,000 people visited the museum today.
b. The Louvre didn't get that many visitors: at most 12,000 people visited the museum today.

In this section I will delve deeper into this evaluative meaning component. In section 8.1 I will explore the nature of the evaluative meaning and argue that it is a conventional implicature. In section 8.2 I will draw a parallel between

[19] Measure phrase equatives such as 'as many as 100' are similar to DNMs in this regard, see Rett (2014) for an elaborate discussion.

modified numerals and adjectives, and I will defend the claim that it is the property of having an open scale that brings about an evaluative meaning.

8.1 Nature of the evaluative meaning

The evaluative meaning component of DNMs is not part of their at-issue content. One way to see this is to consider a dialogue like the one given in (106). It is clearly odd for Sarah to suggest that Angie's remark is untrue based on the fact that she disagrees with the notion that 100 is a high number of guests.

(106) Angie: 'There will be up to 100 people at my birthday party!'
Sarah: '#{No / That's not true}, 100 people is not a lot.'

Non-at-issueness is a property of conversational implicatures, conventional implicatures, and presuppositions. First, let us explore the possibility that, like their upper bound, the evaluativity of DNMs is a conversational implicature. The easiest way to test this is to check whether the evaluative meaning component is defeasible. To this end, let us consider (107) and (108).

(107) a. There will be up to 100 people at my birthday party, #which is not that many.
b. There will be at most 100 people at my birthday party, which is not that many.

(108) a. The mayor got up to 60% of the votes, #which is much less than expected.
b. The mayor got at most 60% of the votes, which is much less than expected.

The continuations in the a-sentences sound very odd, especially compared to to the ones in the b-sentences with *at most*. This suggests that the evaluative meaning component is not a conversational implicature.

Another argument against a conversational implicature analysis of the evaluativity of DNMs is the fact that it is non-detachable. The evaluative meaning is specific to the lexical item *up to* and its crosslinguistic counterparts. We have established that *up to 100* means something like 'at least some and possibly 100' or 'between one and 100 and possibly more'. If we replace *up to* in (109-a) by these expressions, as in (109-b) and (109-c), the evaluativity disappears. Unlike (109-a), (109-b) and (109-c) are merely factual statements about the possible numbers of guests that will be at the party. They do not say anything about how high the speaker thinks this number is.

(109) a. There will be up to 100 people at my birthday party.
b. There will be at least some and possibly 100 people at my birthday party.
c. There will be between one and 100 people at my birthday party, and possibly more.

It seems safe to conclude that the evaluative meaning component is not a conversational implicature. Another possibility that one might consider is that it is a presupposition. However, presupposition plugs like verbs of saying do not plug the evaluativity of DNMs. This is illustrated in (110). Despite the presence of *said*, (110-a) indicates that the speaker thinks that 100 is a high number of guests. This is further corroborated in (110-b). If *said* were a plug for the evaluative meaning component, the first part of (110-b) would not carry the meaning that the speaker believes 100 to be a high number. It would then be perfectly fine for the speaker to add that she thinks 100 is not many. The fact that the addition of the statement that 100 is not a lot is infelicitous suggests that by uttering the first part of (110-b), the speaker is committed to the claim that 100 is a high number.

(110) a. Jacob said that there will be up to 100 people at his party.
b. Jacob said that there will be up to 100 people at his party, #which is really not that many.

The evaluative meaning component is not part of the at-issue meaning of DNMs, it is not defeasible, non-detachable, and it is invariant under presupposition plugs. All of these observations suggest that we are dealing with a conventional implicature. In addition, evaluativity is speaker-oriented in nature. Speaker-orientedness has been argued to be a central component of conventional implicatures (Potts, 2005).

A potential counterargument to the claim that the evaluativity of DNMs is a conventional implicature comes from antibackgrounding. Potts (2005) gives the example in (111) to illustrate this concept.

(111) *Context: it is known that Lance Armstrong survived cancer.*
a. #When reporters interview Lance, a cancer survivor, he often talks about the disease.
b. And most riders know that Lance Armstrong is a cancer survivor.

The idea is that if it is known that Lance Armstrong survived cancer, it is odd to conventionally implicate this information, as in (111-a). It is, however, fine to presuppose it, as in (111-b). This is what is meant by antibackgrounding: if a certain piece of information is part of the common knowledge in the context, uttering it as a conventional implicature leads to redundancy and infelicity.

The antibackgrounding requirement is not satisifed for the evaluativity of DNMs. It is completely fine for a speaker to utter (109-a) in a context where everybody knows that he believes 100 to be a high number of guests.

I believe that the antibackgrounding facts do not dent the argument that the evaluativity of DNMs is a conventional implicature. There are two reasons for this. First, Potts himself concedes that the antibackgrounding test may not be watertight. Citing Steedman's (2000) observations on the topic, Potts remarks that it is possible that presupposition accommodation is an unconscious and easily available process, in which case it is difficult to distinguish between presuppositions and conventional implicatures when it comes to adding new information.

Second, there is an important difference between Pott's example in (111-a) and (109-a). In (111-a), the only information in the appositive is the conventional implicature: Lance Armstrong is a cancer survivor. In (109-a) on the other hand, the conventional implicature of *up to* is but a small part of the information conveyed by *up to*. As we have seen, *up to* also expresses a lower bound and the fact that the predicate possibly holds for all numbers from this lower bound to, in this case, the number 100. Thus, even if it is known that the speaker regards 100 as a high number of guests, the speaker can still use *up to* without this leading to redundancy. For this reason, antibackgrounding may not occur despite the fact that *up to* carries a conventional implicature.

To sum up, there are five reasons to believe that the evaluative meaning component of DNMs is a conventional implicature: non at-issueness, non-defeasibility, non-detachability, invariance under plugs, and speaker-orientedness. In what follows, I draw a parallel between the evaluative property of DNMs and that of open scale adjectives.

8.2 Modified numerals and adjectives

The distinction between AT MOST and DNMs is not unique to the domain of numeral modifiers.[20] In particular, a similar distinction can be drawn in the domain of adjectives. We have seen that the scale structure of AT MOST is as in (112-a) while the scale of DNMs is like (112-b), where n is the number modified by the numeral modifier and BOTS stands for the numeral at the bottom of the contextually relevant scale.

(112) a. $[\ 0\ , \ldots\ , n\ ]$
b. $[\ \textsc{bots}\ , \ldots\ , n\ , \ldots\ )$

The same scales could be used to represent the meaning of absolute, closed scale adjectives on the one hand and relative, open scale adjectives on the other (Kennedy & McNally, 2005).[21] Closed scale adjectives such as *full* and *closed* are like AT MOST: they indicate that there is an upper bound. In the case of AT MOST this upper bound is a number, while in the case of adjectives it is a certain property that holds to a maximal degree, such as the property of being full or closed. This corresponds to the scale in (112-a).

Open scale adjectives such as *tall* or *expensive* are like DNMs. As represented in the scale in (112-b), they signify a lower bound but no upper-bound; they are open-ended. When an item is referred to as expensive, this does not mean that some maximum degree of expensiveness has been reached. There is no upper limit: in principle, one can always imagine something more expensive, or someone taller.[22]

[20] In this section I will use AT MOST to refer to class B numeral modifiers that entail an upper bound such as *at most* and *maximally* and their crosslinguistic counterparts.

[21] I would like to thank Chris Kennedy (p.c.) for making this suggestion.

[22] See Qing (2016) for a discussion on how DNMs are also similar to open scale adjectives in terms of vagueness.

As is well known, the property of evaluativity is also present in the adjectival domain. When not combined with a number, open-scale adjectives such as *tall* convey that the property they denote holds to a degree that is above a contextually determined standard of comparison (often taken to be contributed to the meaning by the morpheme POS in the positive form, cf. Kennedy and McNally (2005); Kennedy (2007)). So, (113) conveys that John's height is above some contextually determined standard. This is what makes a sentence like (113) evaluative.

(113) John is tall.

Another well documented fact is that closed scale adjectives do not have an evaluative meaning. When uttering (114), clearly the speaker is not trying to indicate that the door is closed 'to a high degree'. As the scale is closed, it is not even clear what that would mean: either the door is closed, and then it is closed to a maximal degree, or it is not.

(114) The door is closed.

Thus, there appears to be a correlation between having an open scale and having an evaluative meaning component. The only difference between the evaluative meaning of DNMs and that of open scale adjectives is that in the latter case, the evaluative meaning is part of the at-issue meaning of the adjective. This can be seen in (115). Unlike in the case of DNMs (see (106) above), here the evaluative meaning component can felicitously be denied by saying that the utterance containing it it false.

(115) *Context: John is 1.80m.*
Robert: 'John is tall.'
Jeremy: '{ No / That's not true }, 1.80m is not tall.'

Certain other quantifiers also display the distinction that we observe between DNMs and AT MOST and between open scale and closed scale adjectives. Two examples that come to mind are *a few* versus *few* and *almost all* versus *not quite all*. In (116), (116-a) indicates that the fact that a few students showed up is positive; though there are not many students, the number of students present is still high given the context. (116-b), on the other hand, has no such meaning. On the contrary, it indicates that the number of students that showed up is low.

(116) *Context: not many students are expected to show up to the lecture.*
a. A few students showed up.
b. Few students showed up.

(117) shows the same dichotomy, this time between *almost all* and *not quite all*. Even though the speaker expected all students to show up to the lecture, when she utters (117-a) she still shows herself to be happy about the high number of students present. The opposite is true for (117-b).

(117) *Context: all students are expected to show up to the lecture.*

a. Almost all students showed up.

b. Not quite all students showed up.

These examples demonstrate the same correlation: when an expression only entails a lower bound and not an upper bound, evaluativity can come into play. When an upper bound is expressed, there is no evaluative meaning. (118) and (119) illustrate that it is indeed the case that *a few* and *almost all* set a lower bound but not an upper bound, and that *few* and *not quite all* set an upper bound but not a lower bound.

(118) a. A few of our students showed up, { maybe all of them did / # maybe none of them did }.

b. A few of our students showed up, { #maybe all of them did / maybe none of them did }.

(119) a. Almost all of our students showed up, { maybe all of them did / # maybe none of them did }.

b. Not quite all of our students showed up, { #maybe all of them did / maybe none of them did }.

In the adjectival domain, the evaluative meaning component has been argued to stem from a comparison to a contextually determined standard. I argue that other evaluative expressions also have this meaning component of the degree under discussion being above some standard of comparison. An expression with a DNM such as *up to 100* then not only conveys 'between 1 and 100, and possibly more' but also that the number is above some contextually determined standard. For instance, going back to (109-a), repeated here as (120), the speaker may consider 70 to be a standard number of guests. Then, in addition to the other meaning components of *up to*, he also expresses that the number under discussion is higher than 70. This way, we get evaluativity for DNMs in exactly the same way as is commonly done for adjectives. In addition, it explains the proximity effect we observe for DNMs: when we hear *up to 100* we get the sense that the real number is not somewhere, say, between 20 and 40 but rather somewhere close to 100. By contrast, *at most 100* does not have this proximity effect. If we assume that DNMs convey that the number under discussion is above some contextually determined standard, both proximity and evaluativity can be derived as side effects of this.

(120) There will be up to 100 people at my birthday party.

So far, I have claimed that the standard of comparison is responsible for the evaluativity not only of adjectives but also of DNMs and certain other quantifiers. However, I have not yet answered the question why having an open scale appears to correlate with the meaning being relative to a standard of comparison. While I have no complete answer to this question, I will offer some speculation here.

Having an open scale is essentially the same thing as being upward entailing.

When we compare upward entailing expressions to downward entailing or non-monotone expressions, the former have a weaker meaning than the latter two. For instance, (121-a) denotes an infinity of worlds, as schematised in (122-a), where w_n is shorthand for 'the world in which there are n professors in the room'. As illustrated in (122-b) and (122-c), neither the equivalent to (121-a) in (121-b) with the downward entailing *at most* nor the one in (121-c) with the non-monotone *exactly* denote an infinity of worlds.

(121) a. There are at least five professors in the room.
b. There are at most five professors in the room.
c. There are exactly five professors in the room.

(122) a. $\{w_5, w_6, w_7, ..., \infty\}$
b. $\{w_0, w_1, w_2, w_3, w_4, w_5\}$
c. $\{w_5\}$

We know that there is a mechanism to strengthen the meaning of these weak, upward entailing expressions: scalar implicatures. A scalar implicature can impose a pragmatic upper bound on expressions like *some* or *a few* and, as I have argued, DNMs. Perhaps the standard of comparison is a second mechanism that is available in natural language to strengthen the meaning of upward entailing expressions. Without the standard of comparison, (113) would have the extremely weak meaning that John has a non-zero height, so the standard of comparison comes in to strengthen the truth conditions of the proposition. Some upward entailing/open scale expressions only use the standard of comparison mechanism, such as open scale adjectives. Others use only the implicature mechanism, such as the quantifier *some*, whose lack of evaluative content indicates a lack of relativity to a standard of comparison. Yet others, such as DNMs, use both mechanisms.

Relativity to a standard of comparison, and therefore evaluativity, is of course different from scalar implicatures in these cases in that it is semantic rather than pragmatic — it is a conventional implicature in the case of DNMs and an entailment in the case of adjectives. To be clear, I do not claim that the standard of comparison is a mechanism speakers use to strenghten their utterances but rather that it could result from some principle underlying language that disfavours extremely weak meanings. Relativity to a standard of comparison could be a fossilised conversational implicature in some cases, but this need not be so.

In this section I have drawn a parallel between the evaluative meaning of DNMs and that of open scale adjectives. I have argued that there is a correlation between having an open scale and having an evaluative meaning component and I have claimed that relativity to a standard of comparison is the cause of this for DNMs. This relativity to a standard of comparison explains not only the evaluative but also the proximal meaning component of DNMs.

9 Conclusion

I have argued that DNMs crosslinguistically form a separate class in the domain of numeral modifiers. DNMs crosslinguistically assert a lower bound and implicate an upper bound, quite unlike other class B numeral modifiers that set an upper bound such as *at most* and *maximally*. I have also made the claim that all class B numeral modifiers are subject to a range requirement. These two claims explain that the upper bound but not the lower bound set by DNMs is defeasible, the monotonicity and NPI licensing properties of DNMs, and the bottom-of-the-scale effect. I have shown that an account that incorporates these ideas can be formalised in degree semantics or in inquisitive semantics. Independent evidence for the account comes from downward entailing and other embedded environments, where the implicature of DNMs behaves as predicted.

What I take to be the main contribution of this paper is that different types of modified numerals vary not just with respect to whether or not they give rise to ignorance inferences but also in the strength of the bounds they set. As this mirrors the way these prepositions behave in other areas of the grammar, i.e. in spatial and temporal contexts, this study also indicates that it is important to pay attention to where the lexical items that can be used as numeral modifiers occur besides as numeral modifiers. After all, they are all borrowed from other areas of the grammar: they are superlatives, comparatives, locative and directional prepositions, and adverbs.

Finally, this work draws a parallel between modified numerals and adjectives. The observation that DNMs have an open scale like open scale adjectives allows us to see a broader picture that emerges in the domain of scalar expressions: having an open scale correlates with evaluativity. The considerations presented here thus help us uncover the nature not just of modified numerals but of scalar expressions in general.

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Appendix: DNMs in downward closed inquisitive semantics

It is commonly assumed nowadays that propositions in inquisitive semantics must be *downward closed*: whenever they contain a possibility p, they must also contain all subsets of p. Here I will show that this is a problematic assumption for the present account as well as Coppock and Brochhagen's account.

First, let us remind ourselves of the definition of DNMs, repeated in (123)

(123) $[\![\text{up to}]\!] = \{\lambda n \lambda P.f\{P(m)|\ s \leq m \leq n\}|f \text{ is a choice function}\}$
where $s > 0$ and $s \neq n$

As we have seen, this definition gives us the ten possibilities in (125) for a sentence like (124). That is, it gives us the set of sets of worlds where at least n people died for every n between one and ten.

(124) Up to ten people died in the crash.

(125) $\{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$
where $p_n = \{w_n, w_{n+1}, w_{n+2}, ...\infty\}$

Coppock & Brochhagen's (2013) Exhaustification procedure then removes all but the weakest world from every possibility in (125). Thus, it yields the set of possibilities in (126).

(126) $\{\{w_1\}, \{w_2\}, \{w_3\}, \{w_4\}, \{w_5\}, \{w_6\}, \{w_7\}, \{w_8\}, \{w_9\}, \{w_{10}\}\}$

Now, if we let (125) be downward closed, it not only contains the ten possibilities in (125) but also all subsets of these possibilities. As the possibilities are infinite, they also contain the world in which, for instance, 482 people died. In downward closed inquisitive semantics, all these higher worlds and all combinations of them are also separate possibilities. The three possibilities listed in (127) are now all in the denotation of (124).

(127) $\{w_{849302840}, w_2\}, \{w_{482}\}, \{w_{12}, w_{17}, w_{918}\}$

Let us see what the Exhaustification operation would do here. The definition is repeated in (128).

(128) $\text{EXH}(P,\hat{s}\,) = \{p - q \mid p \in P \wedge q = \{w \mid \exists q' \in \hat{s}\, [w \in q' \wedge p \not\subseteq q']\}\}$
where P is the proposition and $\hat{s}$ is the question under discussion

Now the proposition P is as given in (129), but it is now downward closed, so all of its subsets are also in the denotation. For instance, the denotation also contains all possibilities in (127). The question under discussion $\hat{s}$, as before, is as in (130).

(129) $\text{P} = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7, p_8, p_9, p_{10}\}$

(130) $\hat{s} = \{q'_0, q'_1, q'_2, q'_3, q'_4, ...\}$

As can be seen in (128), Exhaustification yields the set of possibilities $p - q$ such that:

- p is in P
- q is the set of worlds w such that:
  - w is in one of the possibilities q' in $\hat{s}$
  - p is not a subset of q'

Now let us see what this does for the possibility p given in (131).

(131) $\quad$ p $= \{w_{12}, w_{17}, w_{918}\}$

The set of worlds that are in the question under discussion and are not entailed by p are now equivalent to q'_{13}. Therefore, applying Exhaustification to p yields (132).

(132) $\quad p - q = \{w_{12}, w_{17}, w_{918}\} - \{w_{13}, w_{14}, w_{15}, ...\} = \{w_{12}\}$

Applying Exhaustification to the other two possibilities in (127); $\{w_{849302840}, w_2\}$ and $\{w_{482}\}$, yields (133) and (134) respectively.

(133) $\quad p - q = \{w_{849302840}, w_2\} - \{w_3, w_4, w_5, ...\} = \{w_2\}$

(134) $\quad p - q = \{w_{482}\} - \{w_{483}, w_{485}, w_{486}, ...\} = \{w_{482}\}$

So, if we take our sample in (127), Exhaustification returns the three singleton possibilities given in (135).

(135) $\quad \{w_2\}, \{w_{482}\}, \{w_{12}\}$

As can be seen here, the world in which 12 people died and the world in which 482 people died are still in the denotation of (124) despite the Exhaustification procedure. Therefore, Exhaustification fails to yield the upper bound implicature of 10. In the same way that the worlds in the possibilities in (135) did, all worlds w_n for any n will survive Exhaustification. In terms of truth conditions, Exhaustification is a vacuous operation for DNMs in downward closed inquisitive semantics.

This is a problem for the implementation of my account in inquisitive semantics because Exhaustification is required for the upper bound implicature. It is also a problem for Coppock and Brochhagen's account for two reasons. The first reason is that Coppock and Brochhagen take the essential difference between class A and class B modified numerals to be the fact that class B modifiers yield an interactive proposition; a proposition with multiple possibilities. For this reason, the Maxim of Interactive Sincerity generates an ignorance inference for class B modifiers but not for class A modifiers. However, in downward closed inquisitive semantics it is impossible to state that class A modifiers are not interactive. Coppock and Brochhagen claim that a sentence containing a class A modifier such as *more than* has a denotation like the one in (136).

(136) $[\![\text{A hexagon has more than four sides}]\!] = \{\{w_5, w_6, w_7, ...\}\}$

However, we now have to say that (136) also contains all subsets of (136), for instance $\{w_{59}\}$ and $\{w_5, w_{10}\}$. This makes the proposition interactive. The Maxim of Interactive Sincerity yields ignorance inferences for interactive propositions. Thus, in downward closed inquisitive semantics, Coppock and Brochhagen's account would make the incorrect prediction that class A and class B modifiers give rise to the same ignorance effects.

The second reason why assuming downward closure is a problem for Coppock and Brochhagen is that they use Exhaustification to differentiate between bare numerals and numerals modified by *at least* (or 'bare' DPs and DPs modified by *at least*, such as *an assistant professor* vs. *at least an assistant professor*). A sentence with the bare numeral *three* yields a proposition containing a singleton possibility, as in (137), whereas a sentence with a numeral modified by *at least* yields the interactive proposition given in (138).

(137) $\{\{w_3, w_4, w_5, ...\}\}$

(138) $\{p_3, p_4, p_5, ...\}$

As the reader can verify, Exhaustification then yields (139) for (137) and (140) for (138).

(139) $\{\{w_3\}\}$

(140) $\{\{w_3\}, \{w_4\}, \{w_5\}, ...\}$

This explains why bare numerals have an upper bound implicature but numerals modified by *at least* do not. Downward closure does away with this distinction. In downward closed inquisitive semantics, (137) also contains all of its subsets. A sentence with a bare numeral is therefore interactive, and Exhaustification will yield (140), just like in the *at least* case. Thus, the explanation for the difference between bare numerals and numerals modified by *at least* is lost. In addition, bare numerals, like class A modified numerals, are predicted to give rise to ignorance inferences, contrary to fact.

In sum, assuming downward closure leads to two different problems for my and Coppock and Brochhagen's accounts: 1) Exhaustification can no longer yield an upper bound (for DNMs or for bare numerals); and 2) it is no longer possible to distinguish between interactive and non-interactive propositions, so Coppock and Brochhagen's analysis of why Class B modified numerals do but Class A modified numerals and bare numerals do not give rise to ignorance inferences is lost.

This is problematic as downward closed inquisitive semantics is now regarded as the standard framework in inquisitive semantics. I will leave this as an open issue here (but see Ciardelli, Coppock, and Roelofsen (2016) for a new account of modified numerals in downward closed inquisitive semantics).