

# Scope interactions between modals and modified numerals

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## 1 Introduction

**Observation** *At most* lacks the ‘pure permission’ reading that gives the hearer permission to take 0, 1, or 2 biscuits and only has a stronger reading that forbids them from taking more (e.g. Geurts & Nouwen, 2007; Nouwen, 2010; Schwarz, 2011; Coppock & Brochhagen, 2013; Penka, 2014; Kennedy, 2015).

- (1) a. You’re allowed to take fewer than three biscuits.  
b. You’re allowed to take at most two biscuits.

- From (1-b) you can conclude that you cannot take more than two biscuits
- From (1-a) you cannot conclude that you cannot take more than two biscuits

This is a problem for theories that allow the modal to scope over *at most*, which leads to the pure permission reading.

**Main claim** All *class B numeral modifiers* take scope over existential modals.

	Class A	Class B
Lower bound	more than $n$ over $n$	at least $n$ minimally $n$ from $n$ (up) $n$ or more
Upper bound	fewer than $n$ less than $n$ under $n$	at most $n$ maximally $n$ up to $n$ $n$ or fewer $n$ or less

Table 1: Classification of numeral modifiers in English

## Class B numeral modifiers

Class B numeral modifiers: numeral modifiers that give rise to ignorance effects in the absence of an obviating operator (Nouwen, 2010)

- (2) a. I know exactly how many books there are in this bookshop, *class B*  
#and it's { at least / at most } 10,000.  
b. I know exactly how many books there are in this bookshop, *class A*  
and it's { more than / fewer than } 10,000.

When class B numeral modifiers occur with a modal, this effect can disappear.

- (3) You're required to write at least three pages.

Two readings:

1. *Authoritative reading*: is required that you write no fewer than three pages — you write three or more pages in all possible worlds.
2. *Ignorance reading*: I don't know what the minimum number of pages you are required to write is, but the minimum is not lower than three.

Universal quantifiers interact with class B modifiers in a similar way:

- (4) Everyone took at least three biscuits.
1. *Variation reading*: People took different amounts of biscuits, but there was no one who took fewer than three.
  2. *Ignorance reading*: Everyone took a certain number of biscuits. I don't know what that number is, but I know it is not lower than three.

## Roadmap

### Section 2 Previous theories

#### 2.1 Schwarz's neo-Gricean account of class B modifiers

#### 2.2 Penka's modification

### Section 3 The scopal properties of class B modifiers

#### 3.1 A new generalisation

#### 3.2 Consequences of this generalisation

## 2 Previous theories

### 2.1 Schwarz's neo-Gricean account of class B modifiers

**Goal** Account for ignorance effects of class B modifiers in unembedded contexts and for the two readings that arise when they interact with modals

**Method** Quantity implicatures

Denotations:

- (5) a.  $\llbracket \text{at least} \rrbracket = \lambda d_d \lambda P_{\langle d,t \rangle} . \text{MAX}\{n \mid P(n)\} \geq d$   
b.  $\llbracket \text{at most} \rrbracket = \lambda d_d \lambda P_{\langle d,t \rangle} . \text{MAX}\{n \mid P(n)\} \leq d$

where  $\text{MAX}(P) = \iota n . P(n) \wedge \forall n' [P(n') \rightarrow n' \leq n]$  (Heim, 2000)

Horn sets:

- (6) a.  $\{ 1, 2, 3, 4, 5, \dots \}$   
b.  $\{ \text{at least, exactly, at most} \}$   
(7)  $\llbracket \text{exactly} \rrbracket = \lambda d_d \lambda P_{\langle d,t \rangle} . \text{MAX}\{n \mid P(n)\} = d$

Interactions with modals: the two readings (authoritative vs. ignorance) are derived using scope:

- Narrow scope for the modified numeral is supposed to generate an authoritative reading
- Wide scope for the modified numeral is supposed to generate an ignorance reading

#### 2.1.1 Universal modals

- (8) Mary is required to write at least five pages.  
(9) a.  $\Box [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 5 ]$   
b.  $\text{MAX} \{ n \mid \Box [ \text{Mary writes } n \text{ pages} ] \} \geq 5$

Stronger alternatives to (9-a):

- (10) a.  $\Box [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
b.  $\Box [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$

Primary implicatures:

- (11) a.  $\neg B \Box [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
b.  $\neg B \Box [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$   
where 'B' stands for 'the speaker believes that'

Secondary implicatures:

- (12) a.  $B \neg \square [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
 b.  $B \neg \square [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$

The assertion in (9-a) combination with these secondary implicatures yields the authoritative reading: the speaker believes Mary is required to write five or more pages but she is not required to write exactly five and she is not required to write six or more.

The wide scope reading in (9-b) has the stronger alternatives in (13):

- (13) a.  $\text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $\text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} \geq 6$

These alternatives are symmetric: negating both alternatives yields a contradiction to the assertion (Sauerland, 2004). As a result, the primary implicatures cannot be strengthened to secondary implicatures.

Primary implicatures:

- (14) a.  $\neg B \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $\neg B \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} \geq 6$

Impossible secondary implicatures:

- (15) a.  $B \neg \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $B \neg \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} \geq 6$

From the combination of the assertion and the primary implicatures, we can conclude (16).

- (16) a.  $P \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 (follows from (9-b) and (14-b))  
 b.  $P \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} \geq 6$   
 (follows from (9-b) and (14-a))  
 where ‘P’ stands for ‘the speaker considers it possible that’

The speaker considers both (16-a) and (16-b) possible, so we can conclude that she does not know which of these options is true and derive ignorance implicatures:

- (17) a.  $? \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $? \text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} \geq 6$   
 where ‘?’ stands for ‘the speaker does not know whether’

Ignorance reading: The speaker believes that Mary is required to write at least five pages, but she is not sure whether Mary is required to write exactly five pages and she is not sure whether Mary is required to write at least six pages.

The combination of *at most* and a universal modal works the same way.

## How the account is supposed to work:

- Narrow scope for the modified numeral  $\rightarrow$  no symmetry  $\rightarrow$  secondary/scalar implicatures  $\rightarrow$  authoritative reading
- Wide scope for the modified numeral  $\rightarrow$  symmetry  $\rightarrow$  no secondary implicatures  $\rightarrow$  ignorance implicatures  $\rightarrow$  ignorance reading

### 2.1.2 Existential modals — *at least*

(18) Mary is allowed to write at least five pages.<sup>1</sup>

- (19) a.  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 5 ]$   
b.  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \geq 5$

Stronger alternatives to (19-a):

- (20) a.  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
b.  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$

Primary implicatures:

- (21) a.  $\neg B \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
b.  $\neg B \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$

The stronger alternatives are symmetric, so ignorance implicatures are derived:

- (22) a.  $? \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
b.  $? \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$

**Narrow scope reading:** the speaker believes that Mary is allowed to write five or more pages, but she is not sure whether Mary is allowed to write exactly five pages and she is not sure whether Mary is allowed to write six or more pages.

Stronger alternatives to (19-b):

- (23) a.  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} = 5$   
b.  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \geq 6$

Symmetry  $\rightarrow$  ignorance implicatures:

- (24) a.  $? \text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} = 5$   
b.  $? \text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \geq 6$

**Wide scope reading:** the speaker believes that the maximum number of pages Mary is allowed to write is five or more, but she is not sure if the maximum is exactly five or higher than five.

<sup>1</sup>This example is slightly degraded, see McNabb and Penka (2014) for details.

**Problem** As Schwarz admits, the narrow scope reading appears not to be there: (18) seems to convey that there is an upper bound. Schwarz claims that this weaker narrow scope reading is not visible because it is blocked by the stronger wide scope reading.

### 2.1.3 Existential modals — *at most*

(25) Mary is allowed to write at most five pages.

- (26) a.  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \leq 5 ]$   
 b.  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \leq 5$

Stronger alternatives to (26-a):

- (27) a.  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
 b.  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \leq 4 ]$

Symmetry  $\rightarrow$  ignorance implicatures:

- (28) a.  $? \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
 b.  $? \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \leq 4 ]$

**Narrow scope reading:** the speaker believes that Mary is allowed to write five or fewer pages but she is not sure whether Mary is allowed to write exactly five pages and she is not sure whether Mary is allowed to write four or fewer pages. (Non-attested weak reading.)

Stronger alternatives to (26-b):

- (29) a.  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \leq 4$

Symmetry  $\rightarrow$  ignorance implicatures:

- (30) a.  $? \text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $? \text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \leq 4$

**Wide scope reading:** the speaker believes that the maximum number of pages Mary is allowed to write is five or fewer, but she is not sure if the maximum is exactly five or lower than five.

### Three problems

1. The weak narrow scope reading is not attested
2. Existential modals always lead to symmetric alternatives, so it is not possible to derive an authoritative reading for *at most* + an existential modal
3. The only way authoritative readings can be derived in this account is by letting the modified numeral take narrow scope. In the case of *at most* + an existential modal, this can never give rise to the strong upper bound that is needed

**Overview of readings this account generates with existential modals:**

<i>At least</i> + $\diamond$	<i>At most</i> + $\diamond$
<ul style="list-style-type: none"> <li>• 2 ignorance readings</li> <li>• Wide scope ignorance reading: attested</li> <li>• Narrow scope ignorance reading: not attested</li> </ul>	<ul style="list-style-type: none"> <li>• 2 ignorance readings</li> <li>• Wide scope ignorance reading: attested</li> <li>• Narrow scope ignorance reading: not attested</li> <li>• Reading this account does not generate: authoritative reading with a strong upper bound</li> </ul>

Table 2: Readings & problems for Schwarz (2011)

## 2.2 Penka's modification

Penka (2014) posits an account that derives the missing authoritative reading for sentences with *at most* and an existential modal. Her proposal is to decompose *at most* into an antonymising operator ANT and *at least*, as in (31).

$$(31) \quad [\text{at most } n] = [[n \text{ ANT}] \text{ at least}]$$

*At least* and ANT are defined as follows:

$$(32) \quad \begin{array}{l} \text{a. } \llbracket \text{at least} \rrbracket = \lambda d_d \lambda P_{\langle d,t \rangle} . \text{MAX}\{n \mid P(n)\} \geq d \\ \text{b. } \llbracket \text{ANT} \rrbracket = \lambda d_d \lambda P_{\langle d,t \rangle} . \forall d' : d' > d \rightarrow \neg P(d') \end{array}$$

In this decomposition account, a sentence like (33) has the three LFs in (34).

(33) Mary is allowed to write at most five pages.

$$(34) \quad \begin{array}{l} \text{a. } [ \text{ allowed } [ \text{ ANT } 5 [ \lambda d [ \text{ at least } d [ \lambda d' [ \text{ Mary writes } d' \text{ pages } ] ] ] ] ] ] ] ] ] \\ \text{b. } [ \text{ ANT } 5 [ \lambda d [ \text{ at least } d [ \lambda d' [ \text{ allowed } [ \text{ Mary writes } d' \text{ pages } ] ] ] ] ] ] ] ] \\ \text{c. } [ \text{ ANT } 5 [ \lambda d [ \text{ allowed } [ \text{ at least } d [ \lambda d' [ \text{ Mary writes } d' \text{ pages } ] ] ] ] ] ] ] ] \end{array}$$

The LFs in (34-a) and (34-b) have the same truth conditions as (26-a) and (26-b) respectively, but (34-c) is an extra reading. Its truth conditions are given in (35).

$$(35) \quad \begin{array}{l} \forall d' : d' > 5 \rightarrow \neg \diamond [ \text{ MAX}\{ d \mid \text{ Mary writes } d \text{ pages} \} \geq d' ] \\ = \neg \diamond [ \text{ MAX}\{ d \mid \text{ Mary writes } d \text{ pages} \} > 5 ] \end{array}$$

These are the desired strong truth conditions for the authoritative reading: it is not allowed for Mary to write more than five pages.

For the calculation of the implicatures, Penka assumes the following Horn sets:

$$(36) \quad \begin{array}{l} \text{a. } \{ 1, 2, 3, 4, 5, \dots \} \\ \text{b. } \{ \text{ at least, exactly, at most } \} \\ \text{c. } \{ \text{ ANT } , \emptyset \} \\ \text{d. } \{ \diamond , \square \} \end{array}$$

The stronger scalar alternatives derived using these four Horn sets are:

- (37) a.  $\neg\Diamond [ \text{MAX } \{d \mid \text{Mary writes } d \text{ pages} \} > 4 ]$   
 b.  $\Box [ \text{MAX } \{d \mid \text{Mary writes } d \text{ pages} \} = 5 ]^2$

No symmetry  $\rightarrow$  secondary implicatures:

- (38) a.  $B \Diamond [ \text{MAX } \{d \mid \text{Mary writes } d \text{ pages} \} > 4 ]$   
 b.  $B \neg\Box [ \text{MAX } \{d \mid \text{Mary writes } d \text{ pages} \} = 5 ]$

**Reading:** The speaker believes that Mary is not allowed to write more than five pages but she is allowed to submit more than one abstract and she is not required to submit exactly two.

**Penka's solution:**

- Penka generates an extra reading for *at most* on top of Schwarz's readings: the authoritative reading with a strong upper bound
- Penka inherits all of Schwarz's readings, including the non-attested readings where *at least* and *at most* take scope under modals

### 3 Class B modifiers and scope

#### 3.1 A new generalisation

CLAIM: CLASS B MODIFIERS TAKE SCOPE OVER EXISTENTIAL MODALS.

**ARGUMENT 1:** The reading where *at most* takes scope under an existential modal is not attested

(39-b) but not (39-a) has a reading that merely allows taking 0, 1, or 2 biscuits.

- (39) a. You're allowed to take at most two biscuits.  
 b. You're allowed to take fewer than three biscuits.

Kennedy (2015): *At most* does occur in the scope of existential quantifiers. The strong reading is an implicature — the speaker chose to say *at most two* and not *at most three*.

Problem: the upper bound is not cancellable.

- (40) a. You're allowed to take at most two biscuits, #and/but you can also take more.  
 b. You're allowed to take fewer than three biscuits, and/but you can also take more.

<sup>2</sup>Here ANT has been replaced by  $\emptyset$ ,  $\Diamond$  has been replaced by  $\Box$ , and *at least* has been replaced by *exactly*.



If the upper bound is semantic, the only way it can come about is if *at most* takes scope over the existential modal.

**ARGUMENT 2:** The reading where *at least* takes scope under an existential modal is not attested

(41) Mary is allowed to write at least five pages.

Wide scope reading:

(42)  $\text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \geq 5$

Ignorance implicatures:

(43) a.  $? \text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} = 5$   
 b.  $? \text{MAX} \{ n \mid \diamond [ \text{Mary writes } n \text{ pages} ] \} \geq 6$

**Wide scope reading:** the speaker believes that the maximum number of pages Mary is allowed to write is five or more, but she is not sure if the maximum is exactly five or higher than five.

Narrow scope reading:

(44)  $\diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 5 ]$   
 $= \diamond [ \text{Mary writes 5 or more pages} ]$

Ignorance implicatures:

(45) a.  $? \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} = 5 ]$   
 $= ? \diamond [ \text{Mary writes exactly 5 pages} ]$   
 b.  $? \diamond [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \geq 6 ]$   
 $= ? \diamond [ \text{Mary writes 6 or more pages} ]$

**Narrow scope reading:** the speaker believes that Mary is allowed to write five or more pages, but she is not sure whether Mary is allowed to write exactly five pages and she is not sure whether Mary is allowed to write six or more pages.

Schwarz's theory predicts that (46-a) has the same weak truth conditions as (46-b): five or more is allowed.

(46) a. Mary is allowed to write at least five pages.  
 b. Mary is allowed to write more than four pages.

This reading appears not to be there: (46-a) conveys that there is an upper bound to the number of pages Mary is allowed to write (and the speaker does not know where that upper bound is). It does not merely express that five or more is allowed, like (46-b).

Schwarz: the weak narrow scope reading is blocked by the strong wide scope reading.

Test: context where only the weak reading would be felicitous.

Context: At the local supermarket you can buy either only one box of instant formula or you can buy one or more economy packs that contain multiple boxes. You don't know how many boxes are in an economy pack.

<b>Option 1</b>	<b>Option 2</b>
You buy only one box Exactly 1	You buy one or more economy packs 2, 3, 4, 5, 6, 7, ...

You say:

- (47) a. I can buy more than one box of instant formula.  
 b. #I can buy at least two boxes of instant formula.

Intended semantics of (47-b):

- (48)  $\diamond [ \text{MAX} \{ n \mid \text{I buy } n \text{ boxes} \} \geq 2 ]$   
 $= \diamond [ \text{I buy 2 or more boxes} ]$

Intended ignorance implicatures:

- (49) a.  $? \diamond [ \text{MAX} \{ n \mid \text{I buy } n \text{ boxes} \} = 2 ]$   
 $= ? \diamond [ \text{I buy exactly two boxes} ]$   
 b.  $? \diamond [ \text{MAX} \{ n \mid \text{I buy } n \text{ boxes} \} \geq 3 ]$   
 $= ? \diamond [ \text{I buy three or more boxes} ]$

The fact that this sentence is not felicitous in the context indicates that the weak reading that arises when *at least* takes scope under an existential modal is simply not there. By extension, the data point towards the conclusion that lower-bounded class B modified numerals, like their upper-bounded counterparts, take scope over existential modals.

ARGUMENT 3: When class B modifiers are forced to take scope under an existential modal, the resulting sentence is infelicitous

Class B modifiers do take scope under universal modals:

- (50) a. Mary is required to write at most five pages.  
 b.  $\square [ \text{MAX} \{ n \mid \text{Mary writes } n \text{ pages} \} \leq 5 ]$   
 c.  $\text{MAX} \{ n \mid \square [ \text{Mary writes } n \text{ pages} ] \} \leq 5$

The denotation in (50-b) corresponds to the authoritative reading with a strong upper bound; Mary writes between 0 and 5 pages in all permissible worlds. (50-c) corresponds to an ignorance reading that allows Mary to write more than five pages. The authoritative reading is illustrated in (51).

- (51) In most cities, Uber vehicles are required to be at most 10 years old (year 2006 or newer).<sup>3</sup>

Finite clauses are islands for QR (e.g. Fox, 2000):

- (52) a. A woman loves every man.  $\exists > \forall / \forall > \exists$   
 b. A woman noticed that every man had arrived.  $\exists > \forall / *\forall > \exists$

As expected, class B modifiers, like class A modifiers, are felicitous when they cannot QR and are forced to take scope under a universal modal.

- (53) a. ‘Humane Farm Animal Care’ requires that hens have { at least / more than } 1.5 square feet of space.<sup>4</sup>  
 b. New government regulations require that nurses work { at most / less than } 40 hours a week.

However, they cannot be trapped under existential modals, unlike class A modifiers:

- (54) a. New government regulations allow that nurses work { #at least / more than } 40 hours a week.  
 b. ‘Humane Farm Animal Care’ allows that hens have { #at most / less than } 1.5 square feet of space.

**Potential objection 1:** the *it is allowed that* construction is bad for independent reasons.

The same pattern holds in Dutch, in which the *it is allowed that* construction is fine:

- (55) a. Het is nodig dat je { minder / meer } dan vijf pagina’s schrijft.  
 It is necessary that you { fewer / more } than five pages write.  
 ‘It is necessary that you write { fewer / more } than five pages.’  
 b. Het is nodig dat je { minstens / hoogstens } vijf pagina’s  
 It is necessary that you { at least / at most } five pages  
 schrijft.  
 write.  
 ‘It is necessary that you write { at least / at most } five pages.’
- (56) a. Het is toegestaan dat je { minder / meer } dan vijf pagina’s  
 It is permitted that you { fewer / more } than five pages  
 schrijft.  
 write.  
 ‘It is permitted that you write { fewer / more } than five pages.’  
 b. #Het is toegestaan dat je { minstens / hoogstens } vijf pagina’s  
 It is permitted that you { at least / at most } five pages  
 schrijft.  
 write.

<sup>3</sup>Source: <http://www.uberkit.net/blog/23-reasons-why-your-uber-car-cannot-be-used-for-uber/>, last consulted 12-12-2015

<sup>4</sup>Slightly modified, from <http://www.nytimes.com/2008/09/17/dining/17eggs.html?>, last consulted 13-12-2015

‘It is permitted that you write { at least / at most } five pages.’

**Potential objection 2:** *At least* and *at most* in (54) are fine in an echoic context.

- (57) A: Are nurses allowed to work at least 40 hours a week?  
B: Yes, new government regulations allow that nurses work at least 40 hours a week.

This wouldn’t be the first time that something that is normally ruled out is fine in an echoic context, e.g. a PPI being licensed in the scope of negation:

- (58) A: John saw something.  
B: No, he didn’t see something.

In sum: there are both semantic and syntactic arguments that support the claim that class B modifiers always take scope over existential modals.

## 3.2 Consequences of this generalisation

- There are two parameters along which the readings of class B modifiers with modals can vary:
  1. Scope: letting a numeral modifier take scope over or under a modal leads to different truth conditions
  2. Pragmatics: class B modified numerals with modals can give rise to an authoritative reading or an ignorance reading
- It is assumed in the current literature on modified numerals (the neo-Gricean accounts in Büring, 2008; Schwarz, 2011, 2013; Kennedy, 2015 but also accounts in other frameworks such as Nouwen, 2010; Coppock & Brochhagen, 2013) that these two parameters should be linked to one another. Generally, it is said that wide scope leads to an ignorance reading and narrow scope leads to an authoritative reading.
- If class B modifiers always take scope over existential modals but do give rise to an authoritative reading and an ignorance reading (i.e. there is only one possible scope configuration but there are two readings), we cannot maintain that scope is the cause of the two readings we observe, at least for sentences with existential modals.

(59) a. Professor who has full knowledge of what is allowed: ‘Mary is allowed to write at most five pages.’  
b. Mary’s fellow student: ‘I don’t know exactly what the page limit is, but I know Mary is allowed to write at most five pages.’
- For sentences with class B modifiers with universal modals, there are two options:
  1. We account for their ambiguity using scope (as the current accounts do).

2. We account for their ambiguity in a different way as well.

The second option seems preferable as it would lead to a uniform account of sentences with class B modifiers and modals.

- Thus: we need to tease apart scope and the authoritative/ignorance ambiguity. Our account for this ambiguity should not depend on scope.

**Argument for ‘scope-independent’ theory:** We need a scope-independent theory for the ambiguity of these kinds of examples for an independent reason.

Heim-Kennedy generalisation: degree quantifiers can bind their trace across modals but not across nominal quantifiers (Heim, 2000).

This generalisation seems to hold here:

(60) Every student submitted at most two abstracts.

- (61) a. [ At most 2 [  $\lambda d$  [ every student [  $\lambda x$  [  $x$  submits  $d$  abstracts ] ] ] ] ]  
b.  $\text{MAX} \{ n \mid \forall x [ \text{student}(x) \rightarrow x \text{ submits } n \text{ abstracts} ] \} \leq 2$

This says that the maximum number of abstracts such that all students submitted that many abstracts is two or lower, so the number of abstracts submitted by the student who submitted the least amount of abstracts is two or lower.

(61-b) is not a possible reading of (60), so *at most* must have narrow scope:

- (62) a. [ Every student [  $\lambda x$  [ at most 2 [  $\lambda d$  [  $x$  submits  $d$  abstracts ] ] ] ] ]  
b.  $\forall x [ \text{student}(x) \rightarrow \text{MAX} \{ n \mid x \text{ writes } n \text{ pages} \} \leq 2 ]$

Sentences like (60) do have two readings:

- The variation reading: The speaker has full knowledge of the situation (parallel to the authoritative reading) and conveys that different students submitted different numbers of abstracts, and they all submitted two or less.
- The ignorance reading: There is a specific number such that every student submitted that many abstracts. The speaker does not know what that number is, but he knows it is two or less.

Thus, this is another case where only one scope configuration is possible, but where we do find ambiguity between an ignorance and a non-ignorance reading. Therefore, this is another argument for a scope-independent theory of this ambiguity.

## 4 Conclusion

- Claim: All class B modified numerals take scope over existential modals.
- Semantic arguments: we do not get the truth conditions associated with narrow scope readings for *at least* or for *at most*.
- Syntactic argument: when *at least* and *at most* are trapped under an existential modal in a finite clause island, the resulting sentence is infelicitous.
- Consequence 1: *At most* and *at least* do not differ in their scopal properties. Therefore, a decomposition story à la Penka (2014), in which *at most* has different scopal properties than *at least*, does not account for the data we observe.
- Consequence 2: Scope and the authoritative/ignorance ambiguity should be treated as two separate phenomena. More evidence for this claim comes from interactions between modified numerals and universal quantifiers, which also allow only one scope configuration but do give rise to two readings

Questions for future research:

- Why do class B modifiers take scope the way they do?
- How can we account for the ambiguity of sentences with modals and class B modifiers without resorting to scope?

# References

- Büring, D. (2008). The least *at least* can do. In *Proceedings of the west coast conference on formal linguistics* (Vol. 26, p. 114-120).
- Coppock, E., & Brochhagen, T. (2013). Raising and resolving issues with scalar modifiers. *Semantics & Pragmatics*, 6(3), 1-57.
- Fox, D. (2000). *Economy and semantic interpretation*. Cambridge, MA: MIT Press.
- Geurts, B., & Nouwen, R. (2007). At least *et al.*: the semantics of scalar modifiers. *Language*, 83(3), 533-559.
- Heim, I. (2000). Degree operators and scope. In *Proceedings of SALT 10* (p. 40-64). Ithaca, NY: Cornell Linguistics Club.
- Kennedy, C. (2015). A ‘de-Fregean’ semantics (and neo-Gricean pragmatics) for modified and unmodified numerals. *Semantics and Pragmatics*, 8(10), 1-44.
- McNabb, Y., & Penka, D. (2014). *An experimental investigation of superlative modifiers and modals*. Draft.
- Nouwen, R. (2010). Two kinds of modified numerals. *Semantics & Pragmatics*, 3(3), 1-41.
- Penka, D. (2014). ‘At most’ at last. In E. Csipak & H. Zeijlstra (Eds.), *Proceedings of Sinn & Bedeutung 19*.
- Sauerland, U. (2004). Scalar implicatures in complex sentences. *Linguistics & Philosophy*, 27(3).
- Schwarz, B. (2011). *Remarks on class B numeral modifiers*. Handout of workshop *Indefinites and beyond*, Universität Göttingen.
- Schwarz, B. (2013). *At least* and quantity implicature: Choices and consequences. In M. Aloni, M. Franke, & F. Roelofsen (Eds.), *Proceedings of the 19th Amsterdam Colloquium*.

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