Directional prepositions as numeral modifiers

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1 Introduction

Directional numeral modifiers: expressions that can be used as directional prepositions and as numeral modifiers.

(1) a. Mary walked (all the way) up to the counter.
b. You can make up to five copies.

I asked speakers of 15 different languages about the properties of directional numeral modifiers (DNMs) in their language. It turns out that DNMs crosslinguistically share a bundle of properties.

Languages

- Upper-bounded directional numeral modifiers share certain properties in at least the following languages:
  - Danish
  - Dutch
  - English
  - Farsi
  - French
  - German
  - Greek
  - Hebrew
  - Hungarian[1]
  - Italian
  - Polish
  - Romanian
  - Russian
  - Spanish
  - Turkish
- For details, see Blok (2013, 2015).

(2) Polish: **do**
a. Jan idzie do sklepu.
John goes DO the store.

[1]The Hungarian expression I studied is *közel*. It behaves like a DNM in most ways but seems to have a stronger proximal component than other DNMs and is infelicitous with low numbers.

'John goes up to the store.'

b. Dozwolone do pięciu sztuk bagażu.
Allowed DO five items of luggage.
'It is allowed to take up to five items of luggage.'

(3) Greek: **mehri**

a. Perpatisame mehri tin akri tis limnis.
We walked MEHRI the edge of the lake.
'We walked up to the edge of the lake.'

b. Ston anelkistira khorane mehri 5 atoma.
In the elevator fit MEHRI 5 people.
'Up to 5 people can fit in the elevator.'

Main claims:

- In any language, if an upper-bounded numeral modifier is directional, it has the following five properties:
  - Its upper bound is cancellable.
  - Its lower bound is not cancellable.
  - It displays the bottom-of-the-scale-effect.
  - It is not clearly downward monotone.
  - It does not license NPIs.
- The root of all these properties is that the lower bound of directional numeral modifiers is asserted while their upper bound is implicated.

Structure of the talk:

- The A/B distinction
- Schwarz, Buccola, and Hamilton's (2012) ideas about *up to*
- The bounds of DNMs
- An implicature-based account
- Conclusion

2 Nouwen's (2010) A/B distinction

Nouwen: numeral modifiers can be categorised into two classes: those that obligatorily give rise to ignorance effects and those that do not give rise to these effects.

(4) I know exactly how much memory my laptop has...

a. ...and it is {#at most / # at least / # up to} 2GB.

b. ...and it is {more than / less than / under} 2GB.

	Class A	Class B
Lower bound	more than n over n	at least n minimally n from n (up) n or more
Lower and upper bound	between n and n	from n to n
Upper bound	fewer than n less than n under n	at most n maximally n **up to** n n or fewer n or less

Table 1: Classification of numeral modifiers in English

3 Schwarz et al. (2012) on *up to*

Schwarz et al.: *up to* is different from other upper-bounded class B numeral modifiers such as *at most* in the following ways:

- It displays the bottom-of-the-scale effect
- It is non-monotone
- It does not license NPIs

The bottom-of-the-scale effect

Schwarz et al.: *up to* is incompatible with the numeral at the bottom of the scale it quantifies over.

(5) a. At most ten people died in the crash.
b. At most one person died in the crash.

(6) a. Up to ten people died in the crash.
b. #Up to one person died in the crash.

The bottom-of-the-scale element can be higher or lower than 1:

(7) Context: eggs are sold in cartons of six
a. He bought at most six eggs.
b. #He bought up to six eggs.

(8) Context: cakes are sold per slice
a. She bought at most one whole cake.

b. She bought up to one whole cake.

In all languages I looked at, directional numeral modifiers display the bottom-of-the-scale effect.

(9) Danish:

a. Højest én person døde ved sammenstødet.
   At most one person died in the accident.

b. #Op til én person døde ved sammenstødet.
   Up to one person died in the accident.

(10) Spanish:

a. Como mucho una persona murió en el accidente.
   At most one person died in the accident.

b. #Hasta una persona murió en el accidente.
   Up to one person died in the accident.

The bottom-of-the-scale effect is not just a property of *up to* but of DNMs crosslinguistically.

NPI licensing

Schwarz et al.: *up to* does not license NPIs

(11) a. At most three people had ever been in this cave.
b. *Up to three people had ever been in this cave.

(12) a. At most three students give a damn about Pavarotti.
b. *Up to three students give a damn about Pavarotti.

Again, this appears to be a property of directional numeral modifiers crosslinguistically rather than an idiosyncrasy of the English expression *up to*.

(13) Dutch:

a. Er hoeven maximaal vijf studenten te komen.
   There must maximally five students to come.
   'At most five students have to show up.'

b. *Er hoeven tot vijf studenten te komen.
   There must up to five students to come.
   'Up to five students have to show up.'

(14) French:

a. ?Trois personnes au plus ont vu qui que ce soit.
   Three persons maximally have seen anyone.
   'At most three people have seen anyone.'

b. *Jusqu'à trois personnes ont vu qui que ce soit.
   Up to three persons have seen anyone.

If we follow Ladusaw (1979), this suggests that DNMs are either upward entailing or non-monotone.

Monotonicity

Schwarz et al.: *up to* is non-monotone.[2]

(15) a. At most three students smoke. $\models$
b. At most three students smoke cigars.

(16) a. Up to three students smoke. $\not\models$ *(Schwarz et al.)* / ? $\models$ *(my informants)*
b. Up to three students smoke cigars.

My informants rejected neither the entailment pattern in (16) nor the opposite pattern ((16-b) $\models$ (16-a)) in their languages.

Schwarz et al.'s account

Schwarz et al. propose a non-monotone semantics for *up to*. Their semantics for *up to* has two components:

1. It sets an upper bound.
2. It contains a *range requirement*.

(17) Up to ten people died in the crash.

(17) is then taken to mean 1) that according to the epistemic possibilities considered by the speaker, the maximal number of people who died is ten, and 2) that the number of epistemic possibilities considered by the speaker must be at least two.

The range requirement gives rise to ignorance effects and accounts for the bottom-of-the-scale effect.

(18) #Up to one person died in the crash.

(18) is ruled out because the range requirement is not satisfied; the only possibility considered by the speaker is the possibility that one person died.

Two issues:

1. The bottom-of-the-scale effect is not fully accounted for. As the 0-possibility is not ruled out, *up to + BOTS numeral* still expresses two possibilities: the 0-possibility and the BOTS numeral-possibility. There is no violation of the range requirement.

[2]Schwarz et al.'s explanation of this intuition: 'Our intuitions indicate that (16-b) cannot be inferred from (16-a). Specifically, [...] in a scenario where the speaker is sure that exactly one, two or three students smoke, while also being sure that exactly one or two (but not three) students smoke cigars, (16-a) is true and appropriate, while (16-b) is not.' (Schwarz et al., 2012, p.7)

2. A consequence of positing the range requirement for *up to* only means you miss a generalisation when it comes to accounting for ignorance effects of class B modifiers.

4 It's all in the bounds

There are two additional contrasts between DNMs one the one hand and other upper-bounded class B numeral modifiers on the other hand:

- The lower bound of DNMs is strong and cannot be cancelled.
- The upper bound of DNMs is weak and can be cancelled.

Lower bound

(19) a. At most three students will show up to the lecture, if any.
b. #Up to three students will show up to the lecture, if any.

(20) Italian:
a. Ci saranno al massimo cinque studenti al seminario, se non
There will be maximally five students at the seminar, if not
nessuno.
none.
'There will be maximally five students at the seminar, if any.'
b. #Ci saranno fino a cinque studenti al seminario, se non nessuno.
There will be up to vife students at the seminar, if not none.
'There will be up to five students at the seminar, if any.'

(21) Russian:
a. Na seminare budet maksimum 5 studentov, esli tam voodbshe
At seminar will be maximally five students, if there at all
budut studenti.
will be students.
'There will be maximally five students at the seminar, if any.'
b. #Na seminare budet do 5 studentov, esli tam voodbshe budut
At seminar will be up to five students, if there at all will be
studenti.
students.
'There will be up to five students at the seminar, if any.'

Upper bound

(22) a. #Leftovers keep in the refrigerator for at most one week or more.
b. Leftovers keep in the refrigerator for up to one week or more.[3]

(23) a. #At most ten people died in the crash, perhaps even more.

[3]Source: `http://minimalistbaker.com/best-ever-5-minute-microwave-hummus/`, last consulted 03-11-2014

b. Up to ten people died in the crash, perhaps even more.

(24) Romanian:

a. Până la trezeci de persoane au venit la petrecere.
Up to thirty of persons have come to the party.
'Up to thirty people showed up at the party.'

b. De fapt, cred că au venit treizeci şi două de persoane.
In fact, I think that have come thirty-two of persons.
'In fact, I think that thirty-two people showed up.'

(25) a. Cel mult trezeci de persoane au venit la petrecere.
At most thirty of persons have come to the party.
'At most thirty people showed up at the party.'

b. ?De fapt, cred că au venit treizeci şi două de persoane.
In fact, I think that have come thirty-two of persons.
'In fact, I think that thirty-two people showed up.'

(26) Turkish:

a. Partiye 30 kadar insan geldi.
To party 30 up to person came.
'Up to thirty people showed up at the party.'

b. Sanirim 32 insan geldi.
I think 32 person came.
'I think 32 people showed up.'

(27) a. Partiye en cok 30 insan geldi.
To party at most 30 person came.
'At most thirty people showed up at the party.'

b. ?Sanirim 32 insan geldi.
I think 32 person came.
'I think 32 people showed up.'

The fact that these properties hold for DNMs crosslinguistically makes sense given the fact that in spatial and temporal contexts, these expressions also have a defeasible end-point.

(28) Joan worked (from 9am) until 10pm today. She may have even stayed later than that.

(29) Harry ran (from school) all the way up to his house. I think he may even have gone on to run to the football field after that.

Contrasts between DNMs and other upper-bounded class B modifiers:

	DNMs	Other upper-bounded class B
Bottom-of-the-scale effect	Yes	No
NPI licensing	No	Yes
Monotonicity	?	Downward monotone
Lower bound	Not cancellable	Cancellable
Upper bound	Cancellable	Not cancellable

Table 2: Summary of the data

5 An implicature-based account

5.1 Two generalisations

I propose that the following two generalisations explain the contrasts between DNMs and other upper-bounded class B numeral modifiers:

1. The lower bound of DNMs is asserted while their upper bound is implicated.
2. All class B numeral modifiers require quantification over a range of values.

In my account, DNMs convey that the degree predicate holds for an interval on a scale consisting of at least two numbers. The asserted lower bound is contextually determined. There is no maximality operator or other mechanism to set an upper bound in the semantics.

(30) Up to ten people died in the crash.

Thus, (30) conveys the possibilities that [1,...,10] people died in the crash without excluding any other possibilities.

The bottom-of-the-scale effect

The bottom-of-the-scale effect can be accounted for in the same way as in Schwarz et al. (2012): using the bottom-of-the-scale numeral leads to a singleton set of possibilities, which violates the range requirement.

As the current account comprises a range requirement for *all* class B numeral modifiers, it predicts that they all display the bottom-of-the-scale effect. This prediction is borne out. The difference between DNMs and other upper-bounded numeral modifiers is that the former assert a lower bound.

(31) a. #Up to one person died in the crash.
b. #At most zero people died in the crash.

(32) a. Up to two people died in the crash.
b. At most one person died in the crash.

Ignorance effects

Positing a range requirement for *all* class B numeral modifiers also gives you a uniform account of ignorance effects.

NPIs and monotonicity

As DNMs assert a lower bound but not an upper bound, they are predicted to be upward entailing, i.e. (33-a) entails (33-b).

(33) a. Up to three students smoke cigars. $\models$
b. Up to three students smoke.

However, there is a pragmatically derived upper bound. The fact that there is a semantic lower bound but a pragmatic upper bound can blur intuitions on entailment. This explains the observed variation in judgments.

The fact that DNMs are semantically upward entailing is compatible with the fact that they do not license NPIs.

5.2 Additional evidence: the interaction with evaluative adverbs

Evaluative adverbs generally seem to target the assertion of an utterance and not the implicature, as demonstrated in (34).

(34) a. Fortunately, some students attended the wedding. (Nouwen, 2006)
b. Fortunately, the soup is warm.

The speaker of (34-a) is happy that *at least some* students attended the wedding, not that not all students did. Similarly, (34-b) is used to convey that it is a good thing that the soup is *at least warm*, not that it is not hot.

Similarly, while the speaker of (35-a) expresses her joy about the high number of guests that will attend the wedding, the person uttering (35-b) conveys that she is happy that no more than 100 people will be there. This is evidence for the claim that the asserted content of (35-a) is a lower bound while the asserted content of (35-b) is an upper bound.

(35) a. Fortunately, up to 100 people will attend my wedding.
b. Fortunately, at most 100 people will attend my wedding.

Again, this property holds for DNMs crosslinguistically, as illustrated below for Farsi and German.

(36) Farsi:
a. Khoshbakhtane mitoonam ta 5 rooz morakhasi begiram.
Fortunately I can up to five days get time off work.

b. ?Khoshbakhtane un khanandeye eftezah ta 5 (ta) ahang
Fortunately that singer horrible up to 5 (up to) songs
mixanad.
sing.
'Fortunately, that horrible singer will sing up to five songs.'

(37) a. ?Khoshbakhtane mitoonam hade aksar panj rooz morakhasi begiram.
Fortunately I can at most five days get time off work.
b. Khoshbakhtane un khanandeye eftezah hade aksar 5 ta ahang
Fortunately that singer horrible at most 5 to songs
mixanad.
sing.
'Fortunately, that horrible singer will sing at most five songs.'

(38) German:
a. Glücklicherweise kann ich bis zu fünf Tage frei kriegen.
Fortunately can I up to five days off get.
'Fortunately, I can get up to five days off.'
b. ?Glücklicherweise singt dieser schlechte Sänger bis zu fünf Songs.
Fortunately sings that bad singer up to five songs.
'Fortunately, that bad singer will sing up to five songs.'

(39) a. ?Glücklicherweise kann ich maximal fünf Tage frei kriegen.
Fortunately can I maximally five days off get.
'Fortunately, I can get at most five days off.'
b. Glücklicherweise singt dieser schlechte Sänger maximal fünf Songs.
Fortunately sings that bad singer maximally five songs.
'Fortunately, that bad singer will sing at most five songs.'

6 Conclusion

Directional numeral modifiers are crosslinguistically different from non-directional numeral modifiers in that their upper bound is cancellable while their lower bound is not. Assuming that the former is an implicature while the latter is entailed leads to an account of the bottom-of-the-scale effect, monotonicity properties and interactions with evaluative adverbs.

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Appendix

1. Open issues

Proximity

Like *almost* but unlike *at most*, DNMs seem to have a proximal component:

(40) a. Fortunately, almost all my friends will attend my wedding.
b. Fortunately, up to 200 of my friends will attend my wedding.
c. Fortunately, at most 200 of my friends will attend my wedding.

Both (40-a) and (40-b) but not (40-c) suggest the precise number is under but close to all my friends/200 friends. What is the nature of this element of the meaning of DNMs?

Directivity

DNMs seem to presuppose that the number they modify is a high number (a phenomenon referred to as *directivity* in Nouwen, 2010b):

(41) [In the context of a commercial]
a. Discounts of up to 50%!
b. #Discounts of at most 50%!

What is the nature of this element of their meaning? Is it related to the proximal component?

DE contexts

Clear contrast in the scope of negation:

(42) a. I don't think there will be discounts of up to 70%.
→ The highest discount is lower than 70%.
b. I don't think there will be discounts of at most 70%.
→ The highest discount is higher than 70%.

While (42-a) conveys that the highest discount is *lower* than 70%, (42-b) means that the highest discount is *higher* than 70%.[4] This is expected if we take *up to* to convey that the degree predicate holds for a range of numbers on a scale, while *at most* expresses an upper bound. (42-a) thus means that it is not the case that for all numbers on a scale from 1 to 70, there will be discounts of that amount. (42-b) means that the maximum discount is not 70. Negating a maximum is equivalent to expressing higher numbers are among the possibilities.

Less clear contrast in the antecedent of a conditional:

[4]The 14 speakers I asked all agreed with this judgement. These were speakers of Farsi, French, German, Italian, Polish, Romanian, Russian, Spanish, and Turkish.

(43) a. If you received { up to / at most} ten books you did something wrong.
b. If you order { up to / at most} ten books you have to pay a delivery fee.
c. If you order { up to / at most} ten books you get a discount on your next purchase.

	(43-a)		(43-b)		(43-c)		Total	
No contrast; UB	3	23.1%	8	61.5%	5	38.5%	16	41.0%
No contrast; no UB	6	46.2%	1	7.7%	0	0%	7	17.9%
Contrast: no UB for DNM	4	30.8%	2	15.4%	8	61.5%	14	35.9%
Contrast: no UB for AT MOST	0	0%	2	15.4%	0	0%	2	5.1%
Total	13	100%	13	100%	13	100%	39	

Table 3: Judgments on upper bound implicatures in DE environments[5]

Possible explanation: local implicatures (Chierchia, Fox, & Spector, 2009, in press; Spector, 2014).

(44) a. Joe didn't see Mary or Sue; he saw both.
b. It is not just that you *can* write a reply. You must.
c. I dont expect that some students will do well, I expect that all students will.

Ignorance effects

Ignorance readings of class B modifiers generally become optional but not absent when they occur with a modal or a plural, as in (45).

(45) a. Computers of this kind have at most 2GB of memory.
b. John is allowed to bring at most 10 friends.

The sentences in (45) also have a reading where the computers all have the same memory capacity and John is allowed to bring a fixed number of friends, but the speaker does not know what the exact number is. These ignorance readings in contexts with plurals or modals seem less prominent, if not absent, when a DNM is used, as in (46).

(46) a. Computers of this kind have up to 2GB of memory.
b. John is allowed to bring up to 10 friends.

2. The account in inquisitive semantics

Inquisitive semantics

The ideas presented above can be formalised in the framework of inquisitive semantics (e.g. Ciardelli, Groenendijk, & Roelofsen, 2009, 2012), akin to Coppock and Brochhagen (2013). Inquisitive semantics differs from classical semantics in the following way:

[5]Judgments from speakers of Dutch, French, German, Greek, Italian, Romanian, Russian, Spanish, and Turkish.

- In inquisitive semantics, a proposition expresses a set of possibilities. A possibility is a set of worlds (or classical proposition). A proposition thus conveys a set of sets of worlds.
- This allows for a richer notion of meaning: two propositions that comprise the same set of worlds can differ in meaning because the structure of the propositions is different.

Denotation

I propose the semantics in (47) for DNMs.

(47) $[\![\text{up to}]\!] = \{\lambda n \lambda P.f\{P(m)|\ s \leq m \leq n\}|f \text{ is a choice function}\}$
where $s > 0$ and $s \neq n$

Using this definition, the semantics of (48) is as in (54-c).

(48) Up to ten people died in the crash.

(49) $\{f\{\lambda w \exists x[\#x = m \wedge \text{people}(x)(w) \wedge \text{died-in-the-crash}(x)(w)] \mid$
$s \leq m \leq 10\} \mid f \text{ is a choice function}\}$
$= \{\lambda w \exists x[\#x = m \wedge \text{people}(x)(w) \wedge \text{died-in-the-crash}(x)(w)] \mid s \leq m \leq 10\}$

$= \{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}, ..., \mathbf{p_{10}}\}$
where $p_n = \{w_n, w_{n+1}, w_{n+2}, ...\infty\}$

Ignorance effects

Ignorance effects come about through the Maxim of Interactive Sincerity (Coppock & Brochhagen, 2013):

(50) If φ is interactive, then φ is interactive in the speaker's information set

(51) φ is interactive iff $[\![\varphi]\!]$ contains more than one possibility

As a result of the range requirement, every proposition with a DNM is interactive. Thus, the range requirement in combination with the Maxim of Interactive Sincerity generates ignorance effects.

Upper-bound implicature

The structure of (54-c) enables us to derive the upper bound implicature using Coppock and Brochhagen's exhaustification procedure, given in (52).

(52) $\text{EXH}(P,\hat{s}\ ) = \{p - q \mid p \in P \wedge q = \{w \mid \exists q' \in \hat{s}\ [w \in q' \wedge p \not\subseteq q']\}\}$
where P is the proposition and $\hat{s}$ is the question under discussion

This results in the following outcome for (48):

(53) $\text{P} = \{p_1, p_2, ..., p_{10}\}\ (= \{\{w_1, w_2, w_3, ...\}, \{w_2, w_3, w_4, ...\}, ..., \{w_{10}, w_{11}, w_{12}, ...\}\})$
$\hat{s} = \{q'_0, q'_1, q'_2, q'_3, q'_4, ...\}\ (= \{\{w_0, w_1, w_2, ...\}, \{w_1, w_2, w_3, ...\}, ...\})$

$$\begin{aligned}\textsc{exh}(P,\hat{s}\,) &= p_1 - q = p_1 - \{w_2, w_3, w_4, ...\} = \{w_1\}\\ &\quad p_2 - q = p_2 - \{w_3, w_4, w_5, ...\} = \{w_2\}\\ &\quad ...\\ &\quad p_{10} - q = p_{10} - \{w_{10}, w_{11}, w_{12}, ...\} = \{w_{10}\}\\ &\quad = \{\{\mathbf{w_1}\}, \{\mathbf{w_2}\}, ..., \{\mathbf{w_{10}}\}\}\end{aligned}$$

As (53) illustrates, the exhaustivity operator removes all worlds above w_{10} from the informational content, resulting in an implicated upper bound of 10.

Bottom-of-the-scale effect

As the lower bound s of the scale cannot be zero, the existence of at least one element for which the predicate holds is entailed. As shown in (54), the bottom-of-the-scale effect is accounted for.

(54) a. #Up to one person died in the crash.
b. [up to 1 [$\Diamond$ [λn [n-many people died in the crash]]]]
c. $\{f\{\lambda w \exists x[\#x = m \wedge \text{people}(x)(w) \wedge \text{died-in-the-crash}(x)(w)] \mid 1 \leq m \leq 1\} \mid f \text{ is a choice function}\}$
$= \{\lambda w \exists x[\#x = m \wedge \text{people}(x)(w) \wedge \text{died-in-the-crash}(x)(w)] \mid 1 \leq m \leq 1\}$
$= \{\mathbf{p_1}\}$
where $p_n = \{w_n, w_{n+1}, w_{n+2}, ...\infty\}$

The bottom-of-the-scale numeral s is equal to the numeral *up to* modifies n, so the proposition only denotes a single possibility. This is a violation of the range requirement, which explains the infelicity of (54-a).

3. The account in degree semantics

Denotation

In the framework of degree semantics, the meaning of DNMs can be formalised as in (55).

(55) $[\![\text{up to}]\!] = \lambda n \lambda P \forall m \in [s, ..., n] : P(m)$ where $s > 0$ and $s \neq n$.
The degree predicate P holds for all numbers m on a scale from a contextually determined starting point s to the number n; the numeral modified by the DNM. The starting point is higher than 0 and the scale consists of at least two elements (the range requirement).

For instance, (56-a) has the LF in (56-b), using the counting quantifier *many* (Hackl, 2000) as defined in (57). This results in the semantics in (58).

(56) a. Up to 10 people died in the crash.
b. [up to 10 [λn [n-many people died in the crash]]]

(57) $[\![\text{many}]\!] = \lambda n \lambda P \lambda Q . \exists x [\#x = n \wedge P(x) \wedge Q(x)]$

(58) $\forall m \in [1, ..., 10] : \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$

Ignorance effects

As it is, the meaning of modified numerals with a DNM is now equal to the meaning of bare numerals, assuming a monotone semantics of bare numerals: *up to 10* asserts *at least 10* and implicates *no more than 10*. With (Nouwen, 2010a), I assume that a more complex form that conveys the same meaning as a simple form is blocked (in line with the Maxim of Brevity, Grice, 1975). To rescue the structure, a speaker possibility operator is inserted and the sentence is interpreted with respect to the options the speaker holds possible. This can be observed in (59).

(59) a. [up to 10 [$\Diamond$ [λn [n-many people died in the crash]]]]
b. $\forall m \in [1, ..., 10] : \Diamond \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$

In sum, (56-a) means that the speaker considers it possible that one person died, that two people died, ... , and that ten people died.

The insertion of a speaker possibility operator and with the range requirement result in ignorance effects: since there is always a plurality of numbers on the scale, there will always be multiple possibilities in the speaker's mind.

Upper-bound implicature

The upper-bound implicature is calculated as follows. If the speaker utters (56-a) while knowing that 11 ore more people died, the Maxim of Quantity is violated. Hence (60-a) implicates the negation of all alternative propositions with scales ending in numbers above 10, given in (60-b). The combination of (60-a) and (60-b) lead to the statement in (60-c).

(60) a. $\forall m \in [1, ..., 10] : \Diamond \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)] \rightsquigarrow$
b. $\neg \forall m \in [1, ..., 11] : \Diamond \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)] \wedge$
$\neg \forall m \in [1, ..., 12] : \Diamond \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)] \wedge ...$
c. $\forall m \in [11, ..., \infty] : \neg \Diamond \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$

Bottom-of-the-scale effect

As the lower bound s of the scale cannot be zero, the existence of at least one element for which the predicate holds is entailed. As shown in (61), the bottom-of-the-scale effect is accounted for.

(61) a. #Up to one person died in the crash.
b. [up to 1 [$\Diamond$ [λn [n-many people died in the crash]]]]
c. $\forall m \in [1, ..., 1] : \Diamond \exists x [\#x = m \wedge \text{people}(x) \wedge \text{died-in-the-crash}(x)]$ where $s > 0$ and $s \neq n$.

As s is equal to n, the range requirement is clearly violated, which explains the infelicity of (61-a).

4. Corpus study

Bottom-of-the-scale effect

I collected some additional data that support the claim that DNMs cannot be combined with the number at the bottomof the scale they quantify over from the Corpus of Contemporary American English (COCA) (Davies, 2008).

- I looked at the 29 occurrences of *at most one* and the first 35 occurrences of *up to one*.
- There were 23 occurrences of *at most one* where *one* is the bottom-of-the-scale element, such as in (62-a).
- There were no such cases for *up to one*. *One* was not the bottom-of-the-scale element in any of the instances of *up to one*, such as in (62-b).

(62) a. The lesson is that for any group of economic entities to have a unified currency, there can be at most one independent central bank.[6]
b. The failure to do so can result in a misdemeanor conviction with punishment of up to one year in prison.[7]

	Up to one	*At most one*
One is the BOTS element	0%	79.3%
One is not the BOTS element	100%	20.7%

Table 4: Corpus data on the bottom-of-the-scale effect

Cancellability of the upper bound

A small corpus study I conducted lends further support to the claim that the upper bound of DNMs can be cancelled. I looked for the collocates *maybe*, *perhaps*, and *even* in the range between one and nine words after *up to + number* and *at most + number*. The number of cases of upper bound cancellations I found for each search is given in table 5.[8]

	Up to	*At most*
Maybe	6	0
Perhaps	2	0
Even	17	0

Table 5: Corpus data on upper bound cancellation

[6]Source: M. Friedman. (1994). Free-floating anxiety. *National Review, 46*(17), 32-36.

[7]Source: B. Rankin & G. Bonds Staples. (2011). Sex abuse scandal; Ga. looks to beef up abuse law. *Atlanta Journal Constitution.*

[8]The total number of occurrences of *up to + number* was 17,586 and the total number of occurrences ot *at most + number* was 212. The total number of results for each search were as follows (excluding the cases where *up to* is not used as a numeral modifier): *up to n; maybe*: 14, *at most n; maybe*: 1, *up to n; perhaps*: 11, *at most n; perhaps*: 1, *up to n; even*: 80, *at most n; even*: 2.

Examples of upper bound cancellations with *maybe*, *perhaps*, and *even* are given in (63) below.

(63) a. We're talking about up to 20 – or maybe more – $20 billion bled off to Saddam Hussein.[9]

b. Flood waters of up to 20 feet, perhaps higher, had swept the eastern and southern parts of the city seaward of the barricade of wrecked homes and debris thrown up by the storm.[10]

c. Bullets stacked in the barrel fire at rates of up to 60,000 rounds per minute, even a million in certain multi-barrel configurations.[11]

Methodology

I searched for *up to one* and *at most one* in the COCA corpus. There were 518 occurrences of *up to one* and 31 occurrences of *at most one*. Out of the first 100 occurrences of *up to one*, 44 were cases where *up to* is not used as a numeral modifier (e.g. (64)). I excluded these cases.

(64) The very act of asking opens a guy up to one of his greatest fears: public displays of rejection.[12]

Out of the other 56 instances, 21 were cases where *one* is part of a higher number or a fraction, as in (65).

(65) Up to one hundred twenty women will participate in the trials and they're definitely not doing it for the money.[13]

The remaining 35 were cases where *one* is not the bottom-of-the-scale element, as in (66).

(66) The failure to do so can result in a misdemeanor conviction with punishment of up to one year in prison.[14]

Table 6 summarises these results. In the table in section 3, I disregarded the cases were *one* is part of a higher number or fraction.

[9]Source: Interview with Jeff Flake, Neil Cavuto, Fox, 2004.

[10]Source: P. Hughes. (1990). The great Galveston hurricane. *Weatherwise, 43*(4).

[11]Source: E. Adams. (2004). Is This What War Will Come To? Even as the Pentagon struggles with the low-tech reality of war in Iraq, it looks to increasingly bizarre-sounding technology for next-gen fighting systems. On the following pages, five chapters from the Pentagon's sci-fi future. *Popular Science, 264*(6).

[12]Source: *Ask him anything*, Cosmopolitan magazine, September 2008.

[13]Source: CBS News, 28 September 2009.

[14]Source: B. Rankin & G. Bonds Staples. (2011). Sex abuse scandal; Ga. looks to beef up abuse law. *Atlanta Journal Constitution.*

	Up to one	*At most one*
Higher number/fraction (e.g. (65))	21	2
One is the BOTS element	0	6
One is not the BOTS element	35	23
Total	56	31

Table 6: Corpus data on *up to one* and *at most one*