The semantics and pragmatics
of directional numeral modifiers

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1 Introduction

Directional numeral modifiers: expressions that can be used as directional prepositions and as numeral modifiers.

(1)   a.   Mary walked (all the way) up to the counter.
      b.   You can make up to five copies.

I asked speakers of 15 different languages about the properties of directional numeral modifiers (DNMs) in their language. It turns out that DNMs crosslinguistically share a bundle of properties.

> **Languages**
>
> - Upper-bounded directional numeral modifiers share certain properties in at least the following languages:
>
> | | | | |
> |---|---|---|---|
> | – Danish | – French | – Hungarian | – Russian |
> | – Dutch | – German | – Italian | – Spanish |
> | – English | – Greek | – Polish | |
> | – Farsi | – Hebrew | – Romanian | – Turkish |
>
> - For details, see Blok (2013, 2015).

(2)   Polish: **do**

      a.   Jan idzie do sklepu.
           John goes DO the store.
           'John goes up to the store.'

      b.   Dozwolone do pięciu sztuk bagażu.
             Allowed   DO five   items of luggage.
             'It is allowed to take up to five items of luggage.'

(3)   Greek: **mehri**

      a.   Perpatisame mehri tin akri tis limnis.
             We walked   MEHRI the edge of the lake.
             'We walked up to the edge of the lake.'

      b.   Ston   anelkistira khorane mehri 5 atoma.
             In the elevator   fit   MEHRI 5 people.
             'Up to 5 people can fit in the elevator.'

Main claims:

- In any language, if an upper-bounded numeral modifier is directional, it has the following four properties:

    - Its upper bound is cancellable.

    - Its lower bound is not cancellable.

    - It displays the bottom-of-the-scale-effect.

    - It is not clearly downward monotone.

- The root of all these properties is that the lower bound of directional numeral modifiers is asserted while their upper bound is implicated.

Structure of the talk:

- The A/B distinction

- Schwarz, Buccola, and Hamilton's (2012) ideas about *up to*

- The bounds of DNMs

- An implicature-based account

- Conclusion and discussion

2 Nouwen's (2010) A/B distinction

Nouwen: numeral modifiers can be categorised into two classes: those that obligatorily give rise to ignorance effects and those that do not give rise to these effects.

(4)   I know exactly how much memory my laptop has...

      a.   ...and it is {#at most / # at least / # up to} 2GB.
      b.   ...and it is {more than / less than / under} 2GB.

	Class A	Class B
Lower bound	more than n over n	at least n minimally n from n (up) n or more
Lower and upper bound	between n and n	from n to n
Upper bound	fewer than n less than n under n	at most n maximally n **up to** n n or fewer n or less

Table 1: Classification of numeral modifiers in English

3 Schwarz et al. (2012) on *up to*

Schwarz et al.: *up to* is different from *at most* in the following ways:

- It displays the bottom-of-the-scale effect

- It is non-monotone

- It does not license NPIs

The bottom-of-the-scale effect

Schwarz et al.: *up to* is incompatible with the numeral at the bottom of the scale it quantifies over.

(5)   a.   At most ten poeple died in the crash.
       b.   At most one person died in the crash.

(6)   a.   Up to ten people died in the crash.
       b. #Up to one person died in the crash.

The bottom-of-the-scale element can be higher or lower than 1:

(7)   Context: eggs are sold in cartons of six
       a.   He bought at most six eggs.
       b. #He bought up to six eggs.

(8)   Context: cakes are sold per slice
       a.   She bought at most one whole cake.
       b.   She bought up to one whole cake.

In all languages I looked at, directional numeral modifiers display the bottom-of-the-scale effect.

(9)   Hungarian:
   a.   Legfeljebb egy ember halt meg   az autóbalesetben.
         At most   one man   died PREVERB the car accident.
         'At most one person died in the crash.'
   b. #Közel egy ember halt meg   az autóbalesetben.
         Up to one man   died PREVERB the car accident.
         'Up to one person died in the crash.'

(10)   Spanish:
   a.   Como mucho una persona murió en el   accidente.
         At most   one person died in the accident.
   b. #Hasta una persona murió en el   accidente.
         Up to one person   died   in the accident.

The bottom-of-the-scale effect is not just a property of *up to* but of DNMs generally.

NPI licensing

Schwarz et al.: *up to* does not license NPIs

(11)   a.   At most three people had ever been in this cave.
        b.   *Up to three people had ever been in this cave.

(12)   a.   At most three students give a damn about Pavarotti.
        b.   *Up to three students give a damn about Pavarotti.

Again, this appears to be a property of directional numeral modifiers crosslinguistically rather than an idiosyncrasy of the English expression *up to*.

(13)   Dutch:
   a.   Er   hoeven maximaal vijf studenten te komen.
         There must   maximally five students   to come.
         'At most five students have to show up.'
   b.   *Er   hoeven tot   vijf studenten te komen.
         There must   up to five students   to come.
         'Up to five students have to show up.'

(14)   French:
   a.   ?Trois personnes au plus   ont   vu   qui que ce soit.
         Three persons   maximally have seen anyone.
         'At most three people have seen anyone.'
   b.   *Jusqu'à trois personnes ont   vu   qui que ce soit.
         Up to   three persons   have seen anyone.

If we follow Ladusaw (1979), this suggests that DNMs are either upward entailing or non-monotone.

Monotonicity

Schwarz et al.: *up* to is non-monotone.

(15) a. At most three students smoke. $\models$
      b. At most three students smoke cigars.

(16) a. Up to three students smoke. $\not\models$
      b. Up to three students smoke cigars.

> 'Our intuitions indicate that (16-b) cannot be inferred from (16-a). Specifically, [...] in a scenario where the speaker is sure that exactly one, two or three students smoke, while also being sure that exactly one or two (but not three) students smoke cigars, (16-a) is true and appropriate, while (16-b) is not.'

> (Schwarz et al., 2012, p.7)

My informants rejected neither the entailment pattern in (16) nor the opposite pattern in their languages.

Schwarz et al.'s account

Schwarz et al. propose a non-monotone semantics for *up to*. Their semantics for *up to* has two components:

1. It sets an upper bound.

2. It contains a *range requirement.*

(17) Up to ten people died in the crash.

(17) is then taken to mean 1) that according to the epistemic possibilities considered by the speaker, the maximal number of people who died is ten, and 2) that the number of epistemic possibilities considered by the speaker must be at least two.

The range requirement gives rise to ignorance effects and accounts for the bottom-of-the-scale effect.

(18) #Up to one person died in the crash.

(18) is ruled out because the range requirement is not satisfied; the only possibility considered by the speaker is the possibility that one person died.

A consequence of positing the range requirement for *up to* only means you miss a generalisation when it comes to accounting for ignorance effects of class B modifiers.

4   It's all in the bounds

There are two additional contrasts between DNMs one the one hand and other upper-bounded numeral modifiers on the other hand:

- The lower bound of DNMs is strong and cannot be cancelled.

- The upper bound of DNMs is weak and can be cancelled.

Lower bound

(19)   a.   At most three students will show up to the lecture, if any.
          b. #Up to three students will show up to the lecture, if any.

(20)   Italian:

    a.   Ci   saranno al massimo cinque studenti al   seminario, se non
    There will be   maximally five   students at the seminar,   if not
    nessuno.
    none.
    'There will be maximally five students at the seminar, if any.'

    b. #Ci   saranno fino a cinque studenti al   seminario, se non nessuno.
    There will be   up to vife   students at the seminar,   if not none.
    'There will be up to five students at the seminar, if any.'

(21)   Russian:

    a.   Na seminare budet maksimum 5   studentov, esli tam   voodbshe
    At seminar   will be maximally five students, if   there at all
    budut studenti.
    will be students.
    'There will be maximally five students at the seminar, if any.'

    b. #Na seminare budet do   5   studentov, esli tam   voodbshe budut
    At seminar   will be up to five students, if   there at all   will be
    studenti.
    students.
    'There will be up to five students at the seminar, if any.'

Upper bound

(22)   a. #Leftovers keep in the refrigerator for at most one week or more.
          b.   Leftovers keep in the refrigerator for up to one week or more.[1]

(23)   a. #At most ten people died in the crash, perhaps even more.
          b.   Up to ten people died in the crash, perhaps even more.

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[1]Source: `http://minimalistbaker.com/best-ever-5-minute-microwave-hummus/`, last consulted 03-11-2014

(24)   Romanian:

    a.   Până la trezeci de persoane au   venit la petrecere.
           Up to   thirty of persons   have come to the party.
           'Up to thirty people showed up at the party.'

    b.   De fapt, cred   că   au   venit treizeci şi două de persoane.
           In fact, I think that have come thirty-two   of persons.
           'In fact, I think that thirty-two people showed up.'

(25)   a.   Cel mult trezeci de persoane au   venit la petrecere.
           At most thirty of persons   have come to the party.
           'At most thirty people showed up at the party.'

    b. ?De fapt, cred   că   au   venit treizeci şi două de persoane.
           In fact, I think that have come thirty-two   of persons.
           'In fact, I think that thirty-two people showed up.'

(26)   Turkish:

    a.   Partiye   30 kadar insan   geldi.
           To party 30 up to person came.
           'Up to thirty people showed up at the party.'

    b.   Sanirim 32 insan   geldi.
           I think 32 person came.
           'I think 32 people showed up.'

(27)   a.   Partiye   en cok   30 insan   geldi.
           To party at most 30 person came.
           'At most thirty people showed up at the party.'

    b. ?Sanirim 32 insan   geldi.
           I think 32 person came.
           'I think 32 people showed up.'

The fact that these properties hold for DNMs crosslinguistically makes sense given the fact that in spatial and temporal contexts, these expressions also have a defeasible end-point.

(28)   Joan worked (from 9am) until 10pm today. She may have even stayed later than that.

(29)   Harry ran (from school) all the way up to his house. I think he may even have gone on to run to the football field after that.

Contrasts between DNMs and other upper-bounded class B modifiers:		
	DNMs	**Other upper-bounded class B**
Bottom-of-the-scale effect	Yes	No
NPI licensing	No	Yes
Monotonicity	?	Downward monotone
Lower bound	Not cancellable	Cancellable
Upper bound	Cancellable	Not cancellable

Table 2: Summary of the data

5 An implicature-based account

5.1 Two generalisations

I propose that the following two generalisations explain the contrasts between DNMs and other upper-bounded class B numeral modifiers:

1. The lower bound of DNMs is asserted while their upper bound is implicated.

2. All class B numeral modifiers require quantification over a range of values.

In my account, DNMs convey that the degree predicate holds for an interval on a scale consisting of at least two numbers. The asserted lower bound is contextually determined. There is no maximality operator or other mechanism to set an upper bound.

(30)   Up to ten people died in the crash.

Thus, (30) conveys the possibilities that [1,...,10] people died in the crash without excluding any other possibilities.

The bottom-of-the-scale effect

The bottom-of-the-scale effect can be accounted for in the same way as in Schwarz et al. (2012): using the bottom-of-the-scale numeral leads to a singleton set of possibilities, which violates the range requirement.

As the current account comprises a range requirement for *all* class B numeral modifiers, it predicts that they all display the bottom-of-the-scale effect. This prediction is borne out. The difference between DNMs and other upper-bounded numeral modifiers is that the former assert a lower bound.

(31)   a. #Up to one person died in the crash.
         b. #At most zero people died in the crash.

(32)   a. Up to two people died in the crash.
         b. At most one person died in the crash.

Ignorance effects

Positing a range requirement for *all* class B numeral modifiers also gives you a uniform account of ignorance effects.

NPIs and monotonicity

The account predicts that DNMs are upward entailing, but as the pragmatics contributes an upper bound to the meaning, there is another force pulling it towards being downward monotone. This explains why my informants hesitated to reject either pattern.

The fact that DNMs are semantically upward entailing is compatible with the fact that they do not license NPIs.

5.2 DNMs in an inquisitive framework

5.2.1 Inquisitive semantics

I formalise these ideas in inquisitive semantics (e.g. Ciardelli, Groenendijk, & Roelofsen, 2009, 2012), akin to Coppock and Brochhagen (2013). Inquisitive semantics differs from classical semantics in the following way:

- In inquisitive semantics, a proposition expresses a set of possibilities. A possibility is a set of worlds (or classical proposition). A proposition thus conveys a set of sets of worlds.

- This allows for a richer notion of meaning: two propositions that comprise the same set of worlds can differ in meaning because the structure of the propositions is different.

5.2.2 Formalisation

I propose the semantics in (33) for DNMs.

(33) $[\![\text{up to}]\!] = \{\lambda n \lambda P.f\{P(m)|\ s \leq m \leq n\}|f \text{ is a choice function}\}$
where $s > 0$ and $s \neq n$

Using this definition, the semantics of (34) is as in (35).

(34)   Up to ten people died in the crash.

(35) $\{f\{\lambda w \exists x[\#x = m \land \text{people}(x)(w) \land \text{died-in-the-crash}(x)(w)]\ |$
$s \leq m \leq 10\}\ |\ f \text{ is a choice function}\}$
$= \{\lambda w \exists x[\#x = m \land \text{people}(x)(w) \land \text{died-in-the-crash}(x)(w)]\ |\ s \leq m \leq 10\}$

$= \{\mathbf{p_1}, \mathbf{p_2}, \mathbf{p_3}, ..., \mathbf{p_{10}}\}$
where $p_n = \{w_n, w_{n+1}, w_{n+2}, ...\infty\}$

5.2.3 Ignorance effects

Ignorance effects come about through the Maxim of Interactive Sincerity (Coppock & Brochhagen, 2013):

(36)   If φ is interactive, then φ is interactive in the speaker's information set

(37) φ is interactive iff $[\![\varphi]\!]$ contains more than one possibility

As a result of the range requirement, every proposition with a DNM is interactive. Thus, the range requirement in combination with the Maxim of Interactive Sincerity generates ignorance effects.

5.2.4 Upper-bound implicature

The structure of (35) enables us to derive the upper bound implicature using Coppock and Brochhagen's exhaustification operator, given in (38).

(38) $\text{EXH}(P,\hat{s}) = \{p - q \mid p \in P \wedge q = \{w \mid \exists q' \in \hat{s}\, [w \in q' \wedge p \not\subseteq q']\}\}$
         where P is the proposition and $\hat{s}$ is the question under discussion

This results in the following outcome for (34):

(39) $\text{P} = \{p_1, p_2, ..., p_{10}\}\, (= \{\{w_1, w_2, w_3, ...\}, \{w_2, w_3, w_4, ...\}, ..., \{w_{10}, w_11, w_12, ...\}\})$
         $\hat{s} = \{q'_0, q'_1, q'_2, q'_3, q'_4, ...\}\, (= \{\{w_0, w_1, w_2, ...\}, \{w_1, w_2, w_3, ...\}, ...\})$

$$\text{EXH}(P,\hat{s}) = p_1 - q = p_1 - \{w_2, w_3, w_4, ...\} = \{w_1\}$$
$$p_2 - q = p_2 - \{w_3, w_4, w_5, ...\} = \{w_2\}$$
$$...$$
$$p_{10} - q = p_{10} - \{w_{10}, w_{11}, w_{12}, ...\} = \{w_{10}\}$$

$$= \{\{\mathbf{w_1}\}, \{\mathbf{w_2}\}, ..., \{\mathbf{w_{10}}\}\}$$

As (39) illustrates, the exhaustivity operator removes all worlds above w_{10} from the informational content, resulting in an implicated upper bound of 10.

5.3 Additional evidence: the interaction with evaluative adverbs

Evaluative adverbs generally seem to target the assertion of an utterance and not the implicature, as demonstrated in (40).

(40)   a.   Fortunately, some students attended the wedding.   (Nouwen, 2006)
         b.   Fortunately, the soup is warm.

The speaker of (40-a) is happy that *at least some* students attended the wedding, not that not all students did. Similarly, (40-b) is used to convey that it is a good thing that the soup is *at least warm*, not that it is not hot.

Similarly, while the speaker of (41-a) expresses her joy about the high number of guests that will attend the wedding, the person uttering (41-b) conveys that she is happy that no more than 100 people will be there. This is evidence for the claim that the asserted content of (41-a) is a lower bound while the asserted content of (41-b) is an upper bound.

(41)   a.   Fortunately, up to 100 people will attend my wedding.
            b.   Fortunately, at most 100 people will attend my wedding.

Again, this property holds for DNMs crosslinguistically, as illustrated below for Farsi and German.

(42)   Farsi:
        a.   Khoshbakhtane mitoonam ta   5   rooz morakhasi begiram.
               Fortunately   I can   up to five days get time   off work.
        b. ?Khoshbakhtane un   khanandeye eftezah ta   5 (ta)   ahang
               Fortunately   that singer   horrible up to 5 (up to) songs
               mixanad.
               sing.
               'Fortunately, that horrible singer will sing up to five songs.'

(43)   a.   ?Khoshbakhtane mitoonam hade aksar panj rooz morakhasi begiram.
            Fortunately   I can   at most   five days get time   off work.
        b.   Khoshbakhtane un   khanandeye eftezah hade aksar 5 ta ahang
               Fortunately   that singer   horrible at most   5 to songs
               mixanad.
               sing.
               'Fortunately, that horrible singer will sing at most five songs.'

(44)   German:
        a.   Glücklicherweise kann ich bis zu fünf Tage frei kriegen.
               Fortunately   can I   up to five days off get.
               'Fortunately, I can get up to five days off.'
        b. ?Glücklicherweise singt dieser schlechte Sänger bis zu fünf Songs.
               Fortunately   sings that bad   singer up to five   songs.
               'Fortunately, that bad singer will sing up to five songs.'

(45)   a. ?Glücklicherweise kann ich maximal   fünf Tage frei kriegen.
            Fortunately   can I   maximally five days off get.
            'Fortunately, I can get at most five days off.'
        b.   Glücklicherweise singt dieser schlechte Sänger maximal   fünf Songs.
               Fortunately   sings that bad   singer maximally five songs.
               'Fortunately, that bad singer will sing at most five songs.'

6   Conclusion

Directional numeral modifiers are crosslinguistically different from non-directional numeral modifiers in that their upper bound is cancellable while their lower bound is not. Assuming that the former is an implicature while the latter is entailed leads to an account of the bottom-of-the-scale effect, monotonicity properties and interactions with evaluative adverbs.

7   Open issues

Some open issues:

1. Like *almost* but unlike *at most*, DNMs seem to have a proximal component:

   (46)   a.   Fortunately, almost all my friends will attend my wedding.
           b.   Fortunately, up to 200 of my friends will attend my wedding.
           c.   Fortunately, at most 200 of my friends will attend my wedding.

   Both (46-a) and (46-b) but not (46-c) suggest the precise number is under but close to all my friends/200 friends. What is the nature of this element of the meaning of DNMs?

2. DNMs seem to presuppose that the number they modify is a high number:

   (47)   [In the context of a commercial]
           a.   Discounts of up to 50%!
           b. #Discounts of at most 50%!

   What is the nature of this element of their meaning? Is it related to the proximal component?

3. The upper-bound implicature of DNMs seems to survive in a downward entailing context, as in (48) (although not all my informants agreed; some did not get the implicature in DE contexts), which is unexpected in both the (neo-)Gricean (e.g. Grice, 1975; Horn, 1972, 1989) and the grammatical accounts of implicatures (Chierchia, Fox, & Spector, 2009, in press; Spector, 2014).

   (48)   a.   If you order up to ten books you have to pay a delivery fee.
           b.   If you order up to ten books you get a discount on your next purchase.

   However, if the implicature were absent in these contexts, the modified numeral contributes nothing to the meaning of the utterances. That is, if we interpret (48-a) and (48-b) without the implicature, they have the same meaning as (49-a) and (49-b) respectively.

(49)　　a.　If you order books you have to pay a delivery fee.
　　　　b.　If you order books you get a discount on your next purchase.

As was suggested to me by Benjamin Spector (p.c.), a possible explanation of this is the fact that the modified numeral would be vacuous without the implicature.

There is a clear contrast between DNMs and other upper-bounded numeral modifiers when they occur in the scope of negation:

(50)　　a.　I don't think there will be discounts of up to 70%.
　　　　b.　I don't think there will be discounts of at most 70%.

While (50-a) conveys that the highest discount is *lower* than 70%, (50-b) means that the highest discount is *higher* than 70%. This is expected if we take *up to* to convey that the degree predicate holds for a range of numbers on a scale, while *at most* expresses an upper bound. (50-a) thus means that it is not the case that for all numbers on a scale from 1 to 70, there will be discounts of that amount. (50-b) means that the maximum discount is not 70. Negating a maximum is equivalent to expressing higher numbers are among the possibilities.

4. Ignorance readings of class B modifiers generally become optional but not absent when they occur with a modal or a plural, as in (51).

(51)　　a.　Computers of this kind have at most 2GB of memory.
　　　　b.　John is allowed to bring at most 10 friends.

The sentences in (51) also have a reading where the computers all have the same memory capacity and John is allowed to bring a fixed number of friends, but the speaker does not know what the exact number is. These ignorance readings in contexts with plurals or modals seem less prominent, if not absent, when a DNM is used, as in (52).

(52)　　a.　Computers of this kind have up to 2GB of memory.
　　　　b.　John is allowed to bring up to 10 friends.

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